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Effects of Variable Viscosity and Thermal Conductivity on MHD Couette Flow With Heat Transfer in Presence of Constant Heat Source

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ABSTRACT

Effects of variable viscosity and thermal conductivity on an unsteady free convective MHD Couette flow with heat transfer in presence of a constant heat source has been investigated. The governing equations have been expressed in non-dimensional form using non-dimensional variables, constants, and parameters. The resulting boundary value problem has been solved using Crank -Nicolson method based on Gauss- Seidel method. The effects of various parameters on the velocity and temperature profile are presented graphically. The coefficient of skin fraction, rate of heat transfer in form of Nusselt number are also obtained and presented in tabular form. The effects of all the parameters are significant.

Keywords: Heat transfer, MHD, Variable viscosity and thermal conductivity, Couette flow.

INTRODUCTION

The study of the MHD flow is important as the subject has wide application in different branches of engineering sciences. The applications of MHD include direct power generation and the control of hyper velocity vehicles. MHD convection problems are very significant in the fields of stellar and planetary magnetospheres, chemical engineering and electronics.

The first experimental and electrical work of MHD channel flow was undertaken by Hartmann[1] and Hartmann and Lazarus[2]. Many authors have investigated the two dimensional steady MHD convective Poiseuille flow and heat transfer between two porous plates with constant pressure gradient and heat source. Some of them are Makinde and Mhone[3], Ganesh and Krishnambal[4], Choudhury et al.[5].

Ahmed and Kalita [6], Gulabram and Mishra[7] studied the unsteady MHD flow of conducting fluid through porous medium. Ahmed and Talukdar[8] have studied the MHD Couette flow with heat transfer in presence of constant heat source.

In the study of the problems of heat transfer, it is seen that most of the studies are based on constant physical properties of the fluids. However it is well known that these properties may change with temperature. In convection heat transfer takes place under conditions when there is large temperature differences within the fluid, it becomes necessary to consider variable fluid properties in such studies. Many applications of convection in porous medium are provided in an excellent book by Nield and Bejan [9]. The heat transfer related problems through porous medium becomes very interesting when a magnetic field is applied to the flow of an electrically conducting fluid.

Most of the existing analytical studies for this problem, viscosity and thermal conductivity were assumed to be constant. However it is known from the work of Herwig and Gersten [10] and Hazarika and Khound [11] that these properties may change with temperature. To accurately predict the flow and heat transfer rates, it is necessary to take into account the variation of viscosity and thermal conductivity.

Hence the aim of the present work is to study the effects of variable viscosity and thermal conductivity on unsteady MHD free convective heat transfer flow between two horizontal parallel plates which is basically the work of Ahmed[8].

Mathematical Formulation:

We consider a three dimensional unsteady free convection Couette flow of an incompressible, viscous, electrically conducting fluid bounded by two infinite horizontal parallel plates separated by a distance 'L' in presence of applied magnetic field H' and a constant heat source Q . The upper plate is moving with a uniform velocity.

A co-ordinate system is introduced with origin at the lower plate coinciding with the ZX-plane, X- axis being taken horizontally along the main flow direction. The other plate $y=L$ at a distance 'L' from it is subjected to a uniform motion U' along the +ve x-direction and Y-axis is normal to the plates. Let $q'=(u',0,0)$ be the fluid velocity and $H'=(H_0, H_1', 0)$ be the applied magnetic field at (x', y', z') as shown in the figure 1.

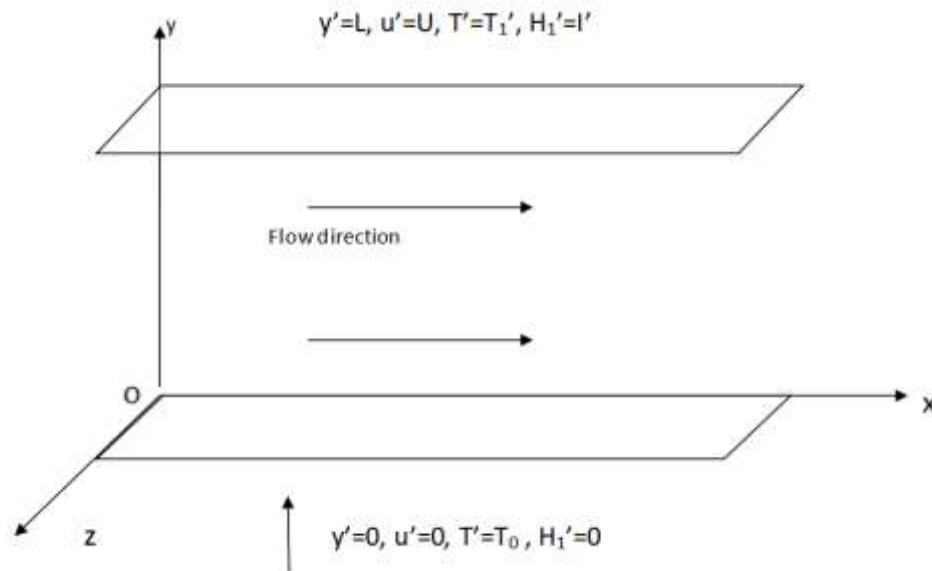


Fig: Flow configuration

The flow is governed by the following equations:

$$\text{Continuity equation: } \frac{\partial u'}{\partial x'} = 0 \Rightarrow u' = u'(y') \quad (1)$$

$$\text{Magnetic equation of continuity: } \frac{\partial H_1'}{\partial x'} = 0 \Rightarrow H_1' = H_1'(y') \quad (2)$$

$$\text{Ampere's law: } J' = -\frac{1}{4\pi} \frac{\partial H_1'}{\partial y'} \quad (3)$$

Neglecting pressure, the momentum equation in x direction,

$$\rho \frac{\partial u'}{\partial t'} = \frac{\partial \mu}{\partial y'} \frac{\partial u'}{\partial y'} + \mu \frac{\partial^2 u'}{\partial y'^2} + \frac{\mu_0 H_0}{4\pi} \frac{\partial H_1'}{\partial y'} \quad (4)$$

Magnetic Induction equation:

$$\frac{\partial H_1'}{\partial t'} = H_0 \frac{\partial u'}{\partial y'} + \frac{1}{4\pi\mu_0\sigma} \frac{\partial H_1'}{\partial y'^2} \quad (5)$$

The Energy equation:

$$\rho c_p \frac{\partial T'}{\partial t'} = \frac{\partial p'}{\partial t'} + \frac{\partial k}{\partial y'} \frac{\partial T'}{\partial y'} + k \frac{\partial^2 T'}{\partial y'^2} + \frac{J'^2}{\sigma} + \phi + Q \quad (6),$$

where ϕ =dissipation fuction

$$= \mu \left(\frac{\partial u'}{\partial y'} \right)^2$$

Now from Ohm's law,

$$J' = \sigma [E' + \mu_0 H_0 u'] \\ = -\frac{1}{4\pi} \frac{\partial H_1'}{\partial y'}$$

Considering $\frac{\partial p'}{\partial t'} = 0$ the energy equation for the problem reduces to,

$$\frac{\partial T'}{\partial t'} = \frac{1}{\rho c_p} \frac{\partial k}{\partial y'} \frac{\partial T'}{\partial y'} + \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{1}{\sigma \rho c_p} \left(-\frac{1}{4\pi} \frac{\partial H_1'}{\partial y'} \right)^2 + \frac{\mu}{\rho c_p} \left(\frac{\partial u'}{\partial y'} \right)^2 + \frac{Q}{\rho c_p} \quad (7)$$

The boundary conditions for this fluid flow problem are:

At $y' = 0$: $u' = 0$, $T' = T_0$, $H_1' = 0$

$y' = L$: $u' = U$, $T' = T_1$, $H_1' = I'$

The fluid viscosity is assumed to be inverse linear function of temperature (Lai and Kulacki [12]) as follows:

$$\frac{1}{\mu} = \frac{1}{\mu_\alpha} [1 + \gamma(T' - T'_\alpha)]$$

Or, $\frac{1}{\mu} = a[T' - T'_r]$, where $a = \frac{\gamma}{\mu}$, $T'_r = T'_\alpha - \frac{1}{\gamma}$, a and T'_r are constants and their values depend upon the reference state and thermal property of the fluid. In general $a > 0$ for liquids and $a < 0$ for gases. T'_r is transformed reference temperature related to viscosity parameter, γ is a constant based on thermal property of the fluid, μ_α is the viscosity $T' = T'_\alpha$.

Let us consider the variation of thermal conductivity as.

$$\frac{1}{k} = \frac{1}{k_\alpha} [1 + \xi(T' - T'_\alpha)]$$

$$= b(T' - T'_c) \text{ where } b = \frac{\xi}{k} \text{ and } T'_c = T'_\alpha - \frac{1}{\xi}, \text{ where } b \text{ and } T'_c \text{ are constants}$$

and their values depend upon the thermal properties of the fluid. T'_c is the transformed reference temperature related to thermal conductivity parameter, ξ is a constant based on thermal property, k_α is the thermal conductivity at $T' = T'_\alpha$.

To normalize the flow model we introduce the following non dimensional parameters:

$$y = \frac{y'}{L}, u = \frac{u'}{U}, t = \frac{t'}{L/U}, T = \frac{T' - T_0}{T_1 - T_0}, H_1 = \frac{H_1'}{H_0}, I = \frac{I'}{H_0}$$

The non-dimensional form of momentum equation is,

$$\frac{\partial u}{\partial t} = \frac{1}{Re} \frac{\theta_r}{(\theta - \theta_r)^2} \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} - \frac{1}{Re} \frac{\theta_r}{(\theta - \theta_r)} \frac{\partial^2 u}{\partial y^2} + \frac{M^2}{Re Rm} \frac{\partial H_1}{\partial y} \quad (1)'$$

where $S = \frac{\mu_0 H_0^2}{4\pi \rho U^2}$ is the ratio of the magnetic energy and kinetic energy in the uniform stream. The number S can be represented in terms of Hartman number (M), Reynolds number (Re) and Magnetic Reynolds number (Rm) as

$$S = \frac{M^2}{Re Rm} \text{ where } M = \mu_0 H_0 \sqrt{\frac{\sigma}{\rho \nu_\alpha}}$$

$$Re = \frac{UL}{\nu_\alpha}$$

$$Rm = 4\pi UL\mu\sigma$$

$$= \frac{UL}{\nu_e}$$

Magnetic Induction equation in non-dimensional form becomes,

$$\frac{\partial H_1}{\partial t} = \frac{\partial u}{\partial y} + \frac{1}{Rm} \frac{\partial^2 H_1}{\partial y^2} \quad (2)'$$

Dimensionless energy equation:

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr Re} \frac{\theta_c}{(\theta - \theta_c)^2} \left(\frac{\partial \theta}{\partial y} \right)^2 - \frac{1}{Pr Re} \frac{\theta_c}{(\theta - \theta_c)} \frac{\partial^2 \theta}{\partial y^2} + \frac{Ec}{Re} \frac{\theta_r}{(\theta - \theta_r)} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{Ec M^2}{(Pm)^2 (Re)^3} \left(\frac{\partial H_1}{\partial y} \right)^2 + Ec Q \quad (3)'$$

and the corresponding boundary conditions are,

$y=0$: $u=0$, $\theta = 0$, $H_1=0$

$y=1$: $u=1$, $\theta = 1$, $H_1=1$

It is now important to calculate the physical quantities of primary interest, which are the local shear stress and the local surface heat transfer.

We can now calculate the local shear stress i.e. skin- friction:

The shear stress at any point in the upper plate is given by

$$\tau' = \mu \left(\frac{\partial u'}{\partial y'} \right)_{y'=L} = \mu \frac{U}{L} \left(\frac{\partial u}{\partial y} \right)_{y=1}$$

The non-dimensional skin friction at the upper plate in the direction of the free stream is given by,

$$\begin{aligned} C_f &= 2 \frac{\mu \left(\frac{\partial u'}{\partial y'} \right)_{y'=L}}{\rho U^2} = 2 \frac{\mu \frac{U}{L} \left(\frac{\partial u}{\partial y} \right)_{y=1}}{\rho U^2} \\ &= 2 \frac{\mu}{\rho U L} \left(\frac{\partial u}{\partial y} \right)_{y=1} \\ &= \frac{2}{Re} \frac{\theta_r}{(1-\theta_r)} \left(\frac{\partial u}{\partial y} \right)_{y=1} \end{aligned}$$

Knowing the temperature field it is interesting to study the rate of heat transfer q^* . The heat flux from the upper plate to the fluid is given by the Fourier law of heat conduction in the form

$$\begin{aligned} q^* &= -k \left(\frac{\partial T'}{\partial y'} \right)_{y'=L} \\ &= -k \frac{T_1 - T_0}{L} \left(\frac{\partial \theta}{\partial y} \right)_{y=1} \end{aligned}$$

The co-efficient of rate of heat transfer from the plate to the fluid in terms of Nusselt is given by

$$\begin{aligned} N_u &= \frac{k}{\rho U C_p (T_1 - T_0)} \left(\frac{\partial T'}{\partial y'} \right)_{y'=L} \\ &= -k \frac{T_1 - T_0}{\rho U L C_p (T_1 - T_0)} \left(\frac{\partial \theta}{\partial y} \right)_{y=1} \\ &= - \left(-\frac{K_\alpha \theta_c}{1 - \theta_c} \right) \frac{T_1 - T_0}{\rho U L C_p (T_1 - T_0)} \left(\frac{\partial \theta}{\partial y} \right)_{y=1} \\ &= \frac{K_\alpha}{\rho C_p} \cdot \frac{1}{v} \frac{v}{U L} \left(\frac{\partial \theta}{\partial y} \right)_{y=1} \\ &= \frac{1}{Pr Re} \left(\frac{\theta_c}{1 - \theta_c} \right) \left(\frac{\partial \theta}{\partial y} \right)_{y=1} \end{aligned}$$

RESULT AND DISCUSSION:

In order to get a clear insight of the physical problem, numerical solutions are displayed with the help of graphs. This enables us to carry out the numerical calculations for the distribution of the velocity and temperature across the boundary for various values of the parameters. The numerical calculations have been

carried by some parameters as $\theta_r = 2$, $\theta_k = 2$, $Ec = 0.05$ $Pr = 0.7$, $Pm = 5$, $Re = 1$, $R_m = 5$.

To determine the accuracy of our present study, we have compared the values of skin- friction as well as Nusselt number with those values reported by Ahmed and Talukdar [8] for different values of Prandtl number Pr . From Table 1 we have observed that the accuracy of present study is very significant than [8].

Figure 1 illustrates the influence of viscosity variation parameter θ_r on velocity distribution. From this figure we have seen that the velocity is decreasing with increasing the viscosity variation parameter. This is because of the fact that the viscous force increases due to increasing values of viscosity. The effect of thermal conductivity variation parameter θ_k on velocity is shown in figure 2. It is clear that the velocity increases with the increasing values of thermal conductivity parameter.

Figure 3 reveals that the velocity decreases on increasing values of M^2 which implies that magnetic field has retarding influence on the fluid velocity. This is due to the fact that the application of a magnetic field to an electrically conducting fluid gives rise to a force, known Lorentz force, which tends to resist the fluid motion.

Figure 4 and figure 5 represent the effect of viscosity variation parameter θ_r as well as thermal conductivity parameter θ_k on temperature distribution of fluid. From figure 4, it is clear that the temperature increases as viscosity variation parameter increases. It is because that as viscosity increases the friction between the layers increases. When friction increases, the heat is generated from the friction on the surface which transferred to the flow. This leads to a rise in the surface temperature and the flow is heated. Also in figure 5, we have observed that temperature decreases on increasing thermal conductivity parameter.

Figure 6 explains the effect of viscous dissipation on temperature profile. Viscous dissipation changes the temperature distribution by playing a role like an energy source, which leads to affect heat transfer rates.

Here the temperature increases with increase in the values of Ec , due to the heat energy that is stored in the liquid and frictional heating. Figure 7 depicts the effect of magnetic Reynolds number R_m on the induced

magnetic field distribution. It is shown that the induced magnetic field increases as R_m increases.

Variation of viscosity parameter θ_r as well as thermal conductivity parameter θ_k on skin-friction and Nusselt number are evaluated numerically in Table 2 and Table 3. It is observed from these tables that skin-friction C_f decreases whereas Nusselt number Nu increases on increasing values of θ_r . On the other hand, the increasing values of θ_c reduces the Nusselt number but enhances the skin friction of fluid.

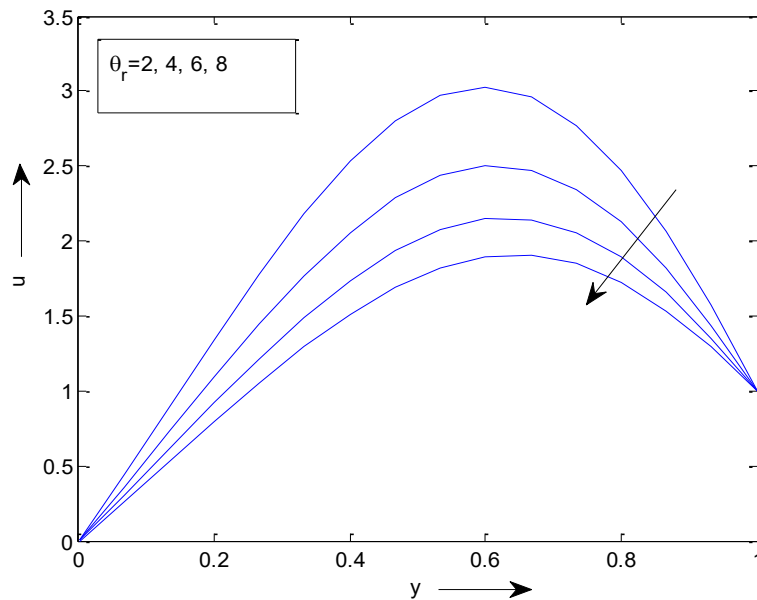


Figure 1: Variation of velocity distribution against y for different values of θ_r

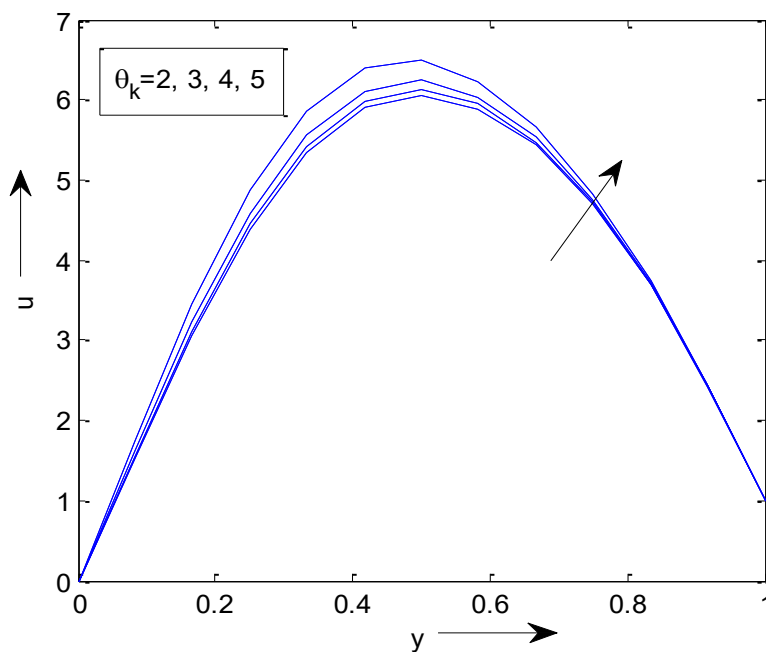


Figure 2: Variation of velocity distribution against y for different values of θ_c

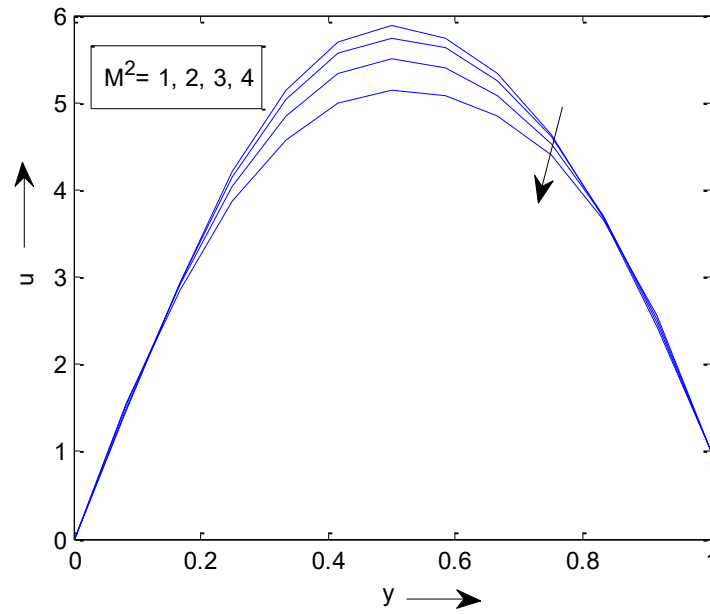


Figure 3: Variation of velocity distribution against y for different values of M^2

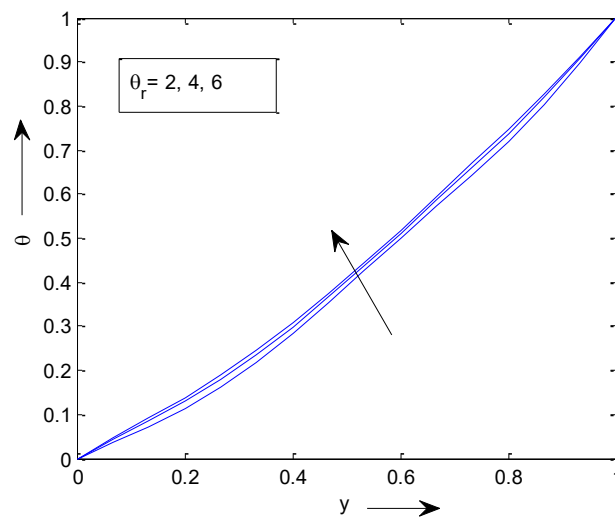


Figure 4: Variation of temperature distribution against y for different values of θ_r

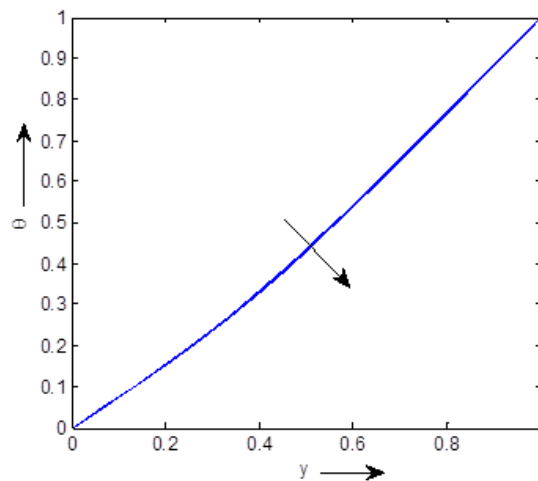


Figure 5: Variation of temperature distribution against y for different values of θ_c

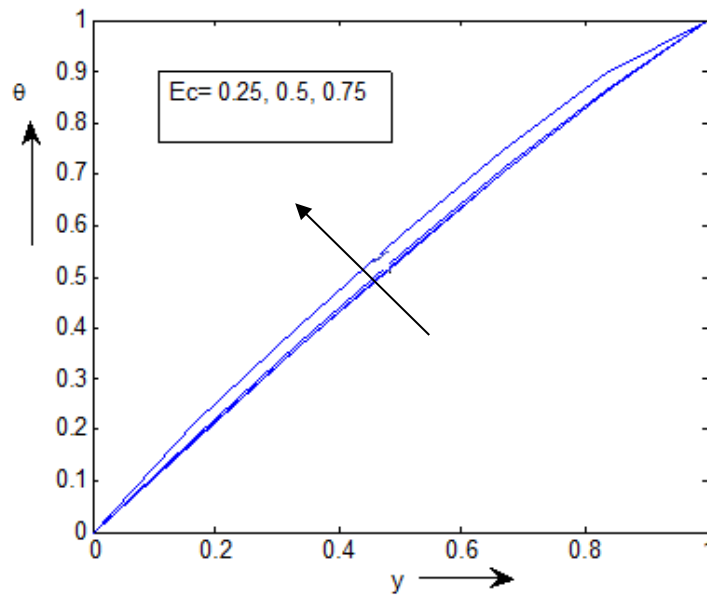


Figure 6: Variation of temperature distribution against y for different values of Ec

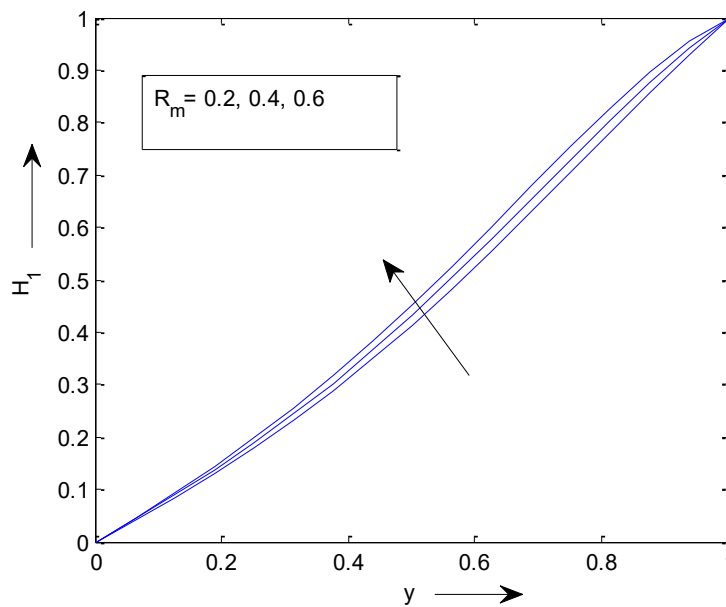


Figure 7: Variation of magnetic field against y for different values of R_m

Table 1: Comparison of C_f and Nu for different values of Pr

Pr	C_f Ahmed and Talukdar[8]	Nu	C_f Present study	Nu
0.25	0.016214	0.293090	0.015506	0.136760
0.5	0.031570	0.976218	0.021469	0.480821
0.75	0.078586	1.010664	0.029528	0.650592

Table 2: Variation of C_f and Nu for various values of θ_r

θ_r	C_f	Nu
2	-0.050011	0.870654
3	-0.078259	1.341651
4	-0.099612	1.415074
5	-0.106942	1.426221

Table 3: Variation of C_f and Nu for various values of θ_c

θ_c	C_f	Nu
2	-0.169366	1.342828
4	-0.165307	1.314573
6	-0.150920	1.285210
8	-0.099612	0.870654

CONCLUSION:

Finally based on the above study we may conclude as below:

1. Magnetic field and viscosity have retarding effect in the flow.
2. The temperature falls due to thermal conductivity, but it increases with increasing viscosity and Eckert number.
3. The induced magnetic field increases as magnetic Reynolds number increases.
4. The coefficient of skin friction (which represents the wall shear stress of fluid) decreases due to viscosity but increases due to thermal conductivity.
5. The increasing values of thermal conductivity reduces the Nusselt number but enhances the skin friction of the fluid.

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