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Effects of Variable Viscosity and Thermal Conductivity on MHD Flow Past A Moving Vertical Plate With Variable Surface Temperature And Concentration in A Porous Medium

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ABSTRACT

This paper investigates the influence of variable viscosity and thermal conductivity on the problem of steady laminar two dimensional magnetohydrodynamic (MHD) boundary layer flow past a moving vertical plate in a porous medium. The temperature and concentration level at the plate surface are assumed to follow a power law type of distribution. Fluid viscosity and thermal conductivity are assumed to vary as an inverse linear function of temperature. The non-linear partial differential equations with prescribed boundary conditions governing the flow are discretized by using Crank-Nicholson formula and the resulting finite difference equations are solved by an iterative scheme based on Gauss-Seidel method by developing computer codes for matlab software. The solution for velocity field, temperature and concentration fields are presented graphically. The skin-friction coefficient at the plate, the Nusselt number and the Sherwood number are presented in tables. The effects of different parameters are also interpreted.

Keywords: porous medium, variable surface temperature and concentration, variable viscosity and thermal conductivity.

INTRODUCTION

Natural convection flows driven by temperature differences are of great interest in a number of industrial applications. Gebhart [5] studied the transient free convection flow past a semi-infinite vertical plate by an approximate method. Bejan and Khair [2] studied free convection heat and mass transfer in a porous medium. Singh and Singh [12] studied the mass transfer effects on the flow past an accelerated vertical plate with constant heat flux. Raptis and Singh [11] studied MHD free convection flow past an accelerated vertical plate. In recent years there has been a growing interest in studying the combined application of MHD flow and porous media. Aldoss et al. [1] investigated combined free and forced convection flow from vertical plate in a porous medium in the presence of magnetic field. In all the above studies the viscosity and thermal conductivity are taken as constants. However these two physical properties of fluids may change significantly with temperature dependent property problem is further complicated by the fact that the properties of different fluids behave differently with temperature. Kafoussias and Williams [9] investigated the effect of temperature dependent viscosity on free-forced convective laminar boundary layer flow past a vertical isothermal plate. Hossain and Munir [7] analysed a two dimensional mixed convection flow of a viscous incompressible fluid of temperature dependent viscosity past a vertical plate. MHD flows and heat transfer have become more important in recent years because of its varied applications in agriculture, engineering and petroleum industries. The free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field has been studied by Elbashbeshy [3]. The heat and mass transfer problems from different geometries embedded in porous media has many practical application in industrial and technological fields. A numerical solution of the transient free convection MHD flow of an incompressible viscous fluid past a semi-infinite inclined plate with variable surface heat and mass flux was discussed by Ganesan and Palani [4]. Combined heat and mass transfer effects on a moving vertical cylinder were investigated by Takhar et al. [13] by using a Crank- Nicholson type implicit finite difference scheme. Jvaherdeh et al. [8] studied the natural convection heat and mass transfer in MHD fluid flow past a moving

vertical plate with variable surface temperature and concentration in a porous medium. However the impact of variable viscosity and thermal conductivity of a MHD free convection flow past a vertical plate embedded in porous medium has received a little attention. Hence an attempt has made to study the effects of variable viscosity and thermal conductivity of the work of Javaherdeh et al. [8].

Formulation of the problem:

We consider a two dimensional free convection steady laminar heat and mass transfer flow of viscous incompressible electrically conducting fluid past a vertical moving plate in a porous medium where a uniform magnetic field is applied in a direction perpendicular to the fluid flow. Figure shows the physical model and co-ordinate system of the problem. The $\bar{x}$ -axis runs along the vertical plate in the direction of flow and $\bar{y}$ -axis is perpendicular to it. The moving plate is under variable temperature boundary condition ($\bar{T}_w = \bar{T}_\infty + bx^n$) and variable concentration ($\bar{c}_w = \bar{c}_\infty + ax^m$).

Our investigation is restricted to the following assumptions:

- (1) The fluid properties are assumed to be constant except for the fluid viscosity and thermal conductivity which are assumed to vary as an inverse linear function of temperature.
 - (2) A uniform magnetic field of strength B_0 is applied normal to the plate.
 - (3) There is no chemical reaction between the fluid and the diffusing species.
 - (4) Plate temperature $\bar{T}_w$ is variable with $\bar{T}_w > \bar{T}_\infty$, $\bar{T}_\infty$ is the free stream temperature assumed constant.
- Under these boundary layer assumptions, the governing equations of the flow are,

Equation of continuity:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad \dots(1)$$

Momentum equations:

$$\left[\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right] = \frac{1}{\rho} \left[\frac{\partial \mu}{\partial \bar{y}} \frac{\partial \bar{u}}{\partial \bar{y}} + \mu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right] + g\beta(\bar{T} - \bar{T}_\infty) + g\beta(\bar{c} - \bar{c}_\infty) - \frac{\nu}{\rho K} \bar{u} + \frac{\bar{F}_m}{\rho} \quad \dots(2)$$

where $\bar{F}_m$ is magneto hydrodynamic force and is calculated as

$$\bar{F}_m = \bar{F}_{em} = \bar{J} \times \bar{B}_0 = \sigma [\bar{q} \times \bar{B}_0] \times \bar{B}_0 = -\sigma \bar{u} B_0^2$$

The energy equation:

$$\rho C_p \left[\bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} \right] = \left[\frac{\partial k}{\partial \bar{y}} \frac{\partial \bar{T}}{\partial \bar{y}} \right] + k \left[\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right] + \mu \left[2 \left(\frac{\partial \bar{u}}{\partial \bar{x}} \right)^2 + \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 \right] \quad \dots(3)$$

Species concentration equation:

$$\bar{u} \frac{\partial \bar{c}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{c}}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}} \left(D \frac{\partial \bar{c}}{\partial \bar{y}} \right) \quad \dots(4)$$

Here μ is the co-efficient of viscosity of the fluid, k is the co-efficient of thermal conductivity of the fluid, C_p is the specific heat at constant pressure and $\sigma, \bar{T}, \rho, \bar{c}, D$ are the electrical conductivity, temperature, density, concentration of the fluid inside the boundary layer and the co-efficient of mass diffusivity of the fluid respectively.

The boundary conditions are as follows:

$$\begin{aligned} \bar{y} = 0 : \bar{u} &= \bar{u}_p, \bar{v} = 0, \bar{T} = \bar{T}_w = \bar{T}_\infty + bx^n, \bar{c} = \bar{c}_w = \bar{c}_\infty + ax^m \\ \bar{y} \rightarrow \infty : \bar{u} &\rightarrow 0, \bar{T} \rightarrow \bar{T}_\infty, \bar{c} \rightarrow \bar{c}_\infty, \end{aligned} \quad \dots(5)$$

Introduce the following non-dimensional quantities and parameters in order to express the governing equations (1-4) and boundary conditions (5) in dimensionless form,

$$\begin{aligned} x &= \frac{\bar{x} u_p}{v_\infty}, y = \frac{\bar{y} u_p}{v_\infty}, V = \frac{\bar{v}}{u_p}, U = \frac{\bar{u}}{u_p}, \theta = \frac{\bar{T}_w - \bar{T}}{\bar{T}_w - \bar{T}_\infty}, \phi = \frac{\bar{c}_w - \bar{c}}{\bar{c}_w - \bar{c}_\infty} \\ p_r &= \frac{\nu}{\alpha}, Sc = \frac{\nu}{D}, Gr = \frac{g\beta(\bar{T}_w - \bar{T}_\infty) u_p^3}{\nu^3}, N = \frac{\beta(\bar{c}_w - \bar{c}_\infty)}{\beta(\bar{T}_w - \bar{T}_\infty)}, k_p = \frac{v_\infty^2}{\rho K u_p^2}, M = \frac{v_\infty \sigma B_0^2}{\rho u_p^2} \end{aligned} \quad \dots(6)$$

The fluid viscosity is assumed to be inverse linear function of temperature (Lai and Kulacki [10]) as below:

$$\frac{1}{\mu} = \frac{1}{\mu_{\infty}} [1 + \gamma(\bar{T} - \bar{T}_{\infty})] \quad (7)$$

or $\mu = \frac{1}{b(\bar{T} - \bar{T}_r)}$, where $b = \frac{\gamma}{\mu_{\infty}}$, $\bar{T}_r = \bar{T}_{\infty} - \frac{1}{\gamma}$. Here μ_{∞} is the viscosity at $\bar{T} = \bar{T}_{\infty}$, b and $\bar{T}_r$ are constants and their values depend upon the reference state and the thermal property of the fluid. In general $b > 0$ for liquids and $b < 0$ for gases. $\bar{T}_r$ is transformed reference temperature related to viscosity parameter, γ is a constant based on thermal property of the fluid.

The dimensionless temperature $\theta(n)$ is defined as $\theta(n) = \frac{\bar{T}_w - \bar{T}}{\bar{T}_w - \bar{T}_{\alpha}}$

$$\mu = \mu_{\infty} \left(\frac{1 - \theta_r}{\theta - \theta_r} \right)$$

where, $\theta_r = \frac{\bar{T}_w - \bar{T}_r}{\bar{T}_w - \bar{T}_{\alpha}}$

It is also important to note that θ_r is negative for liquid and positive for gases when $(\bar{T}_w - \bar{T}_{\alpha})$ is positive (Lai and Kulacki).

Again let us consider the variation of thermal conductivity as [6]:

$$\frac{1}{k} = \frac{1}{k_{\alpha}} [1 + \xi(\bar{T} - \bar{T}_{\alpha})] \quad (8)$$

or, $k = \frac{1}{a(\bar{T} - \bar{T}_c)}$,

where, $a = \frac{\xi}{k_{\alpha}}$, $\bar{T}_c = \bar{T}_{\alpha} - \frac{1}{\xi}$ and $\bar{T}_c$ are constants and their values depend upon the reference state and thermal properties of the fluid. $\bar{T}_c$ is the transformed reference temperature related to thermal conductivity parameter, ξ is a constant based on thermal property. k_{α} is thermal conductivity at $\bar{T} = \bar{T}_{\alpha}$

$$\Rightarrow k = k_{\alpha} \left(\frac{1 - \theta_c}{\theta - \theta_c} \right)$$

where, $\theta_c = \frac{\bar{T}_w - \bar{T}_c}{\bar{T}_w - \bar{T}_{\alpha}}$

The non-dimensional form of continuity equation is:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (9)$$

Momentum equation:

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{1 - \theta_r}{(\theta - \theta_r)} \frac{\partial \theta}{\partial Y} \frac{\partial U}{\partial Y} + \left(\frac{1 - \theta_r}{\theta - \theta_r} \right) \frac{\partial^2 U}{\partial Y^2} + Gr(1 - \theta) + GrN(1 - \phi) - \left(\frac{1 - \theta_r}{\theta - \theta_r} \right) UK_p - MU \quad (10)$$

Dimensionless energy equation:

$$U \frac{\partial \theta}{\partial X} - \frac{n(1 - \theta)}{X} U + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \left[- \frac{1 - \theta_c}{(\theta - \theta_c)} \left(\frac{\partial \theta}{\partial Y} \right)^2 + \left(\frac{1 - \theta_c}{\theta - \theta_c} \right) \frac{\partial^2 \theta}{\partial Y^2} \right] + Ec \left(\frac{1 - \theta_r}{\theta - \theta_r} \right) \left[2 \left(\frac{\partial U}{\partial X} \right)^2 + \left(\frac{\partial U}{\partial Y} \right)^2 \right] \quad (11)$$

Dimensionless species concentration equation:

$$U \frac{\partial \phi}{\partial X} - \frac{m(1 - \phi)}{X} U + V \frac{\partial \phi}{\partial Y} = \frac{1}{Sc} \left[- \frac{1 - \theta_r}{(\theta - \theta_r)} \frac{\partial \theta}{\partial Y} \frac{\partial \phi}{\partial Y} + \left(\frac{1 - \theta_c}{\theta - \theta_c} \right) \frac{\partial^2 \phi}{\partial Y^2} \right] \quad (12)$$

The corresponding boundary conditions are:

$$Y = 0: U = 1, V = 0, \theta = 0, \phi = 0$$

$$Y \rightarrow \infty: U = 0, \theta = 1, \phi = 1$$

Coefficient of skin-friction, Nusselt number and Sherwood number:

The shear stress at the plate, heat flux and mass flux are presented given by Newton's law of viscosity, Fourier's law of conduction and Fick's law in the following form:

$$\tau_x = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}, \quad h_x = -k \left. \frac{\partial T}{\partial y} \right|_{y=0} \quad \text{and} \quad h_D = -D \left. \frac{\partial C}{\partial y} \right|_{y=0}$$

The skin friction coefficient, rate of heat transfer in terms of Nusselt number and rate of mass transfer in terms of Sherwood number are defined by

$$\left. \begin{aligned} C_f &= \frac{\tau_x}{\rho_\infty u_0^2} = \frac{\theta_r - 1}{\theta_r} \left. \frac{\partial U}{\partial Y} \right|_{Y=0} \\ Nu &= \frac{x h_x}{k(T_w - T_\infty)} = -\frac{\theta_c - 1}{\theta_c} \left. \frac{\partial \theta}{\partial Y} \right|_{Y=0} \\ Sh &= \frac{x h_D}{D(C_w - C_\infty)} = -\frac{\theta_r - 1}{\theta_r} \left. \frac{\partial \varphi}{\partial Y} \right|_{Y=0} \end{aligned} \right\} \quad (13)$$

RESULT AND DISCUSSION:

In order to get a clear insight of the physical problem, numerical solutions are displayed with the help of graphs. This enables us to carry out the numerical calculations for the distribution of the velocity and temperature across the boundary for various values of the parameters. The numerical calculations have been carried by some parameters as

$$\theta_r = 2, \theta_k = 2, Ec = 0.05, Pr = 0.7, n = 1, m = 1, Sc = .22, M = .5.$$

Table 1 gives a comparison between the earlier works of Javaherdeh et al. [8] with our present study. In earlier investigation of [8] viscosity and thermal conductivity were assumed constant but in our study we assume the viscosity and thermal conductivity are variable. From the table it is seen that coefficient of skin friction, Nusselt number and Sherwood number decrease with the increase of Pr

Figure 1 displayed the effect of viscosity variation parameter θ_r on velocity distribution. It is clear from this figure that the velocity decreases with increasing values of viscosity variation parameter. This is because of the fact that the viscous force increases due to increasing values of viscosity. The effect of thermal conductivity variation parameter θ_c on velocity is shown in figure 2. We have seen that the velocity increases with the increasing values of thermal conductivity parameter.

Figure 3 reveals that the velocity decreases on increasing values of magnetic parameter M which implies that magnetic field has retarding influence on the fluid velocity. This is due to the fact that the application of a magnetic field to an electrically conducting fluid gives rise to a force, known Lorentz force, which tends to resist the fluid motion.

Figure 4 and figure 5 represent the effect of viscosity variation parameter θ_r as well as thermal conductivity parameter θ_c on temperature distribution of fluid. From figure 4, it is observed that the temperature increases as viscosity variation parameter increases. It is because that as viscosity increases the friction between the layers increases. When friction increases, the heat is generated from the friction on the surface which transferred to the flow. This leads to a rise in the surface temperature and the flow is heated. Also in figure 5, we have observed that temperature decreases on increasing thermal conductivity parameter.

Figure 6 explains the effect of viscous dissipation on temperature profile. Viscous dissipation changes the temperature distribution by playing a role like an energy source, which leads to affect heat transfer rates.

Here the temperature increases with increase in the values of Ec , due to the heat energy that is stored in the liquid and frictional heating.

Figure 7 and figure 8 depict the effect of viscosity variation parameter θ_r and Schmidt number Sc on concentration distribution. From figure 7 we observed that concentration profile decreases with increasing values of θ_r . From figure 8, it can be concluded that concentration profiles of both phases decrease as Sc increases. Since the Schmidt number is the ratio of the momentum to the mass diffusivity. Therefore Schmidt number is the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic and concentration boundary layers. As the Schmidt number increases the concentration decreases.

Variation of viscosity parameter θ_r as well as thermal conductivity parameter θ_c on skin-friction, Nusselt number and Sherwood number are evaluated numerically in Table 2 and Table 3. It is observed from these

tables that coefficient of skin-friction C_f and Sherwood number decrease whereas Nusselt number Nu

increases on increasing values of θ_r . On the other hand, the increasing values of θ_c reduces the Nusselt number but enhance the coefficient of skin friction and Sherwood number of fluid.

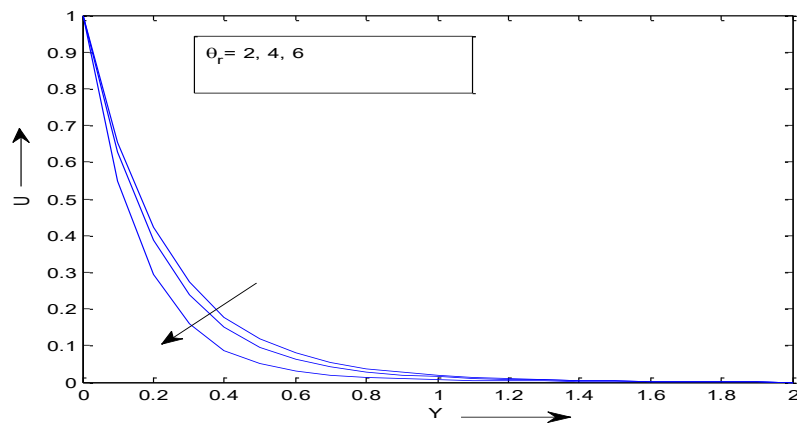


Figure 1: Variation of velocity distribution against Y for different values of θ_r

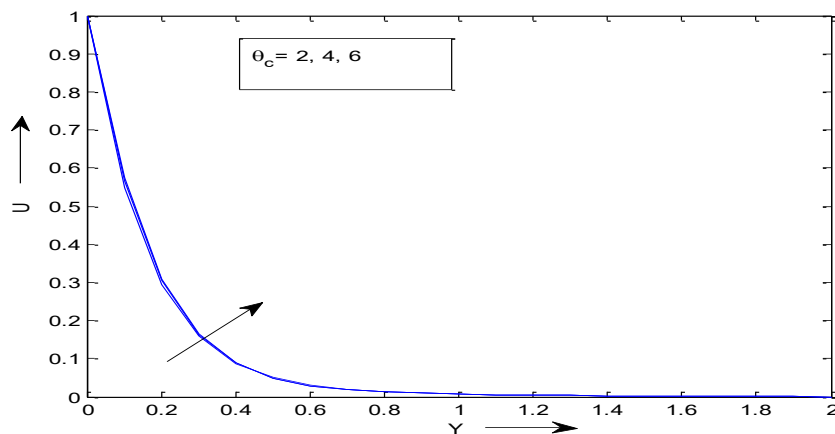


Figure 2: Variation of velocity distribution against Y for different values of θ_c

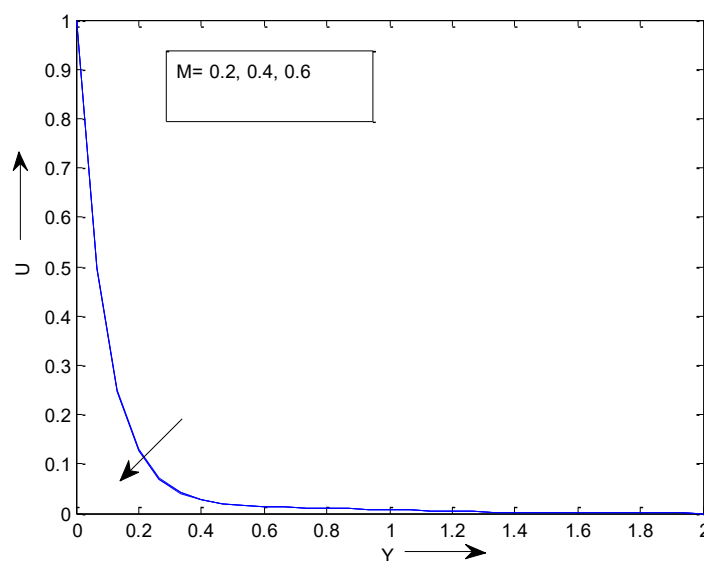


Figure 3: Variation of velocity distribution against Y for different values of M

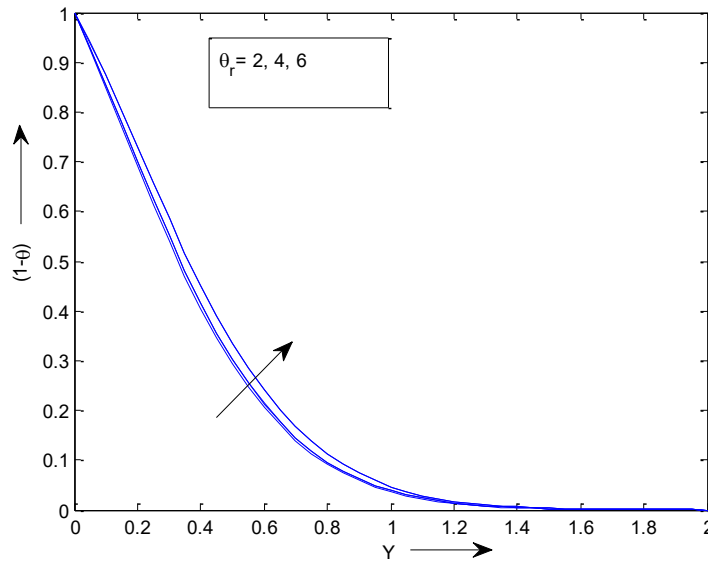


Figure 4: Variation of temperature distribution against Y for different values of θ_r

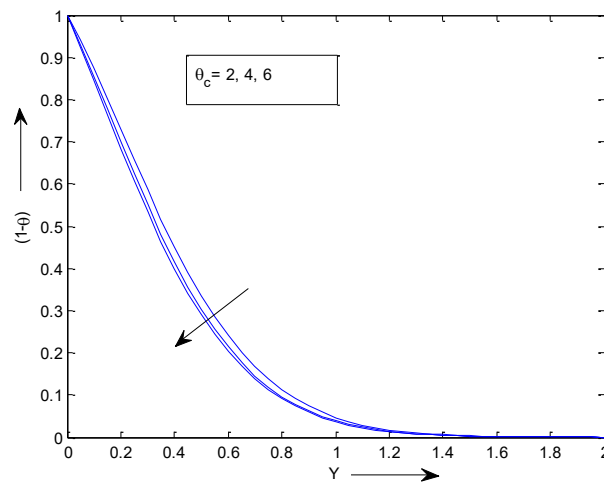


Figure 5: Variation of temperature distribution against Y for different values of θ_c

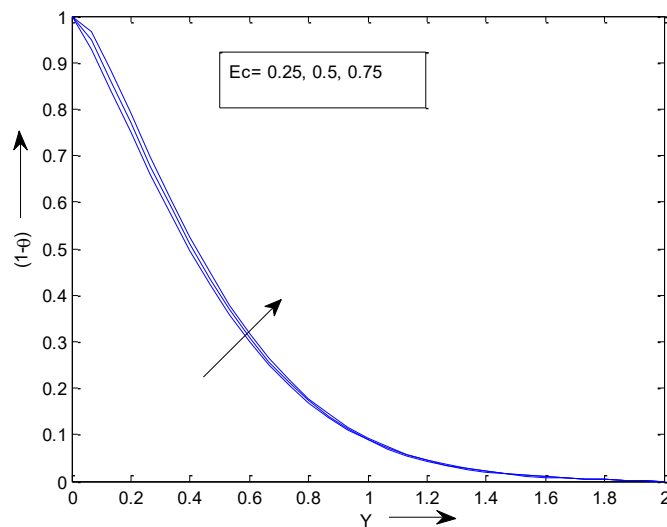


Figure 6: Variation of temperature distribution against Y for different values of Ec

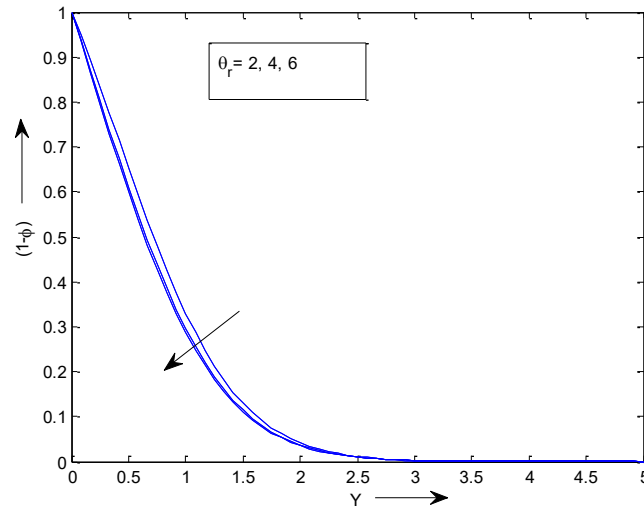


Figure 7: Variation of concentration distribution against Y for different values of θ_r

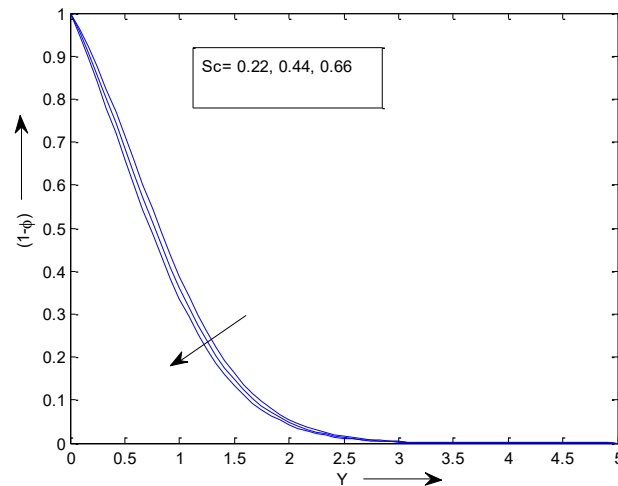


Figure 8: Variation of concentration distribution against Y for different values of Sc

Table 1: Comparison of C_f , Nu and Sh for different values of Pr

Pr	C_f	Nu	Sh	C_f	Nu	Sh
	Present study			Previous study [8]		
0.25	0.217452	0.19811	0.001645	0.234527	0.30307	0.00529
0.5	0.217275	0.19990	0.003545	0.234351	0.30327	0.00529
0.75	0.217087	0.19575	0.005826	0.234175	0.30347	0.005291

Table 2: Variation of C_f , Nu and Sh for various values of θ_r

θ_r	C_f	Nu	Sh
2	-0.70991	-0.29515	0.005335
3	-0.71265	-0.28821	0.005332
4	-0.71723	-0.28423	0.005331
5	-0.72732	-0.28171	0.005330

Table 3: Variation of C_f , Nu and Sh for various values of θ_c

θ_c	C_f	Nu	Sh
2	-0.217126	-0.14433	0.005331
3	-0.158375	-0.15285	0.005354
4	-0.138732	-0.16711	0.005363
5	-0.128901	-0.19585	0.005368
6	-0.123001	-0.28423	0.005371

CONCLUSION:

Finally based on the above study we may conclude as below:

Magnetic field and viscosity have retarding effect in the flow.

The temperature falls due to thermal conductivity, but it increases with increasing viscosity and Eckert number.

The coefficient of skin friction (which represents the wall shear stress of fluid) decreases due to viscosity but increases due to thermal conductivity.

The increasing values of thermal conductivity reduce the Nusselt number but enhance the skin friction of the fluid.

Concentration decreases as Schmidt number as well as viscosity increase.

Sherwood number decreases with the increasing values of viscosity parameter.

NOMENCLATURE:

B_o magnetic field intensity

$\bar{c}$ concentration of the fluid

c_p specific heat at constant pressure

D mass diffusion co-efficient

g acceleration due to gravity

Gr Grashof number

h_x local heat transfer co-efficient

h_d local mass transfer co-efficient

k_p permeability of the porous media

k thermal conductivity

m exponent in power law variation of wall temperature

n exponent in power law variation of wall concentration

M magnetic field parameter

Nu_x local Nusselt number

Pr Prandtl number

Sc Schmidt number

Sh_x local Sherwood number

$\bar{T}$ fluid temperature

$\bar{u}_p$ moving plate velocity

X, Y dimensionless x and y co-ordinates

U, V dimensionless velocity components in X,Y directions

θ dimensionless temperature

φ dimensionless concentration

subscripts :

w condition at the wall

∞ condition at free-stream

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