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On New Class of Continuous Functions in Tri - βg^* Functions in Tri - Topological Spaces

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ABSTRACT

The purpose of this paper is to introduce strongly and perfectly tri - βg^* Continuous Maps, totally tri - βg^* continuous functions and slightly tri - βg^* continuous functions are investigated. Additionally, we relate and compare these functions with some other functions in tri - topological spaces.

Keywords: Strongly tri - βg^* continuous functions, perfectly tri - βg^* continuous functions, totally tri - βg^* continuous and slightly tri - βg^* continuous.

1. INTRODUCTION

In 2016, C. Dhanapakyam and K. Indirani [5] introduced βg^* Closed sets in topological spaces, C. Dhanapakyam and Dr. K. Indirani [7] introduced βg^* closed sets in Bitopological spaces and C. Dhanapakyam and R. Sharmila [8] introduced $(X, \tau_1, \tau_2, \tau_3)$ Closed sets in tri - topological spaces. K. Bala deenaarasi, L. Jeya-sudha and V. Maheswari [4] introduced tri - g^* continuous functions in tri - topological spaces and C. Dhanapakyam and R. Sharmila [9] introduced tri - βg^* Continuous in tri - topological spaces. RC Jain [9] introduced the concept of totally continuous functions and slightly continuous for topological spaces. In this paper we introduced and investigate a new class of functions called strongly tri - βg^* continuous and slightly tri - βg^* continuous functions are investigated and obtained.

2. PRELIMINARIES

Throughout this paper $(X, \tau_1, \tau_2, \tau_3)$, $(Y, \sigma_1, \sigma_2, \sigma_3)$ and $(Z, \eta_1, \eta_2, \eta_3)$ or X, Y, Z represent non - empty tri - topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(X, \tau_1, \tau_2, \tau_3)$, tri - $\text{cl}(A)$ and tri - $\text{int}(A)$ denote the tri - closure and the tri - interior of A respectively. The power set of X is denoted by $P(X)$. If A is tri - βg^* open and tri - βg^* closed, then it is said to be tri - βg^* clopen.

Definition 2.1. A subset A of a tri - topological space X is said to be a tri - βg^* closed [8] if $\text{tri-gcl}(A) \subseteq U$, $A \subseteq U$, U is tri - semi pre open (β tri - open) in X .

Definition 2.2. A subset A of a tri - topological space X is said to be a tri - β open [8] if $A \subseteq \text{tri-cl}(\text{tri-int}(A))$.

Definition 2.3. A function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is called strongly tri - continuous [4] if $f^{-1}(O)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$ for each subset O in $(Y, \sigma_1, \sigma_2, \sigma_3)$.

Definition 2.4. A function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is called tri - βg^* continuous [9] if $f^{-1}(O)$ is a tri - βg^* open set of $(X, \tau_1, \tau_2, \tau_3)$ for every tri - open set O of $(Y, \sigma_1, \sigma_2, \sigma_3)$.

Definition 2.5. A function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is called a perfectly tri - continuous [4] if $f^{-1}(O)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$ for every tri - open set O in $(Y, \sigma_1, \sigma_2, \sigma_3)$.

Definition 2.6. A function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is called a totally tri - continuous [4] if $f^{-1}(V)$ is tri - clopen set in X for each tri - open set V of Y .

Definition 2.7. A function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is called a slightly tri - continuous [4] if the inverse image of every tri - clopen set in Y is tri - open set in X .

Definition 2.8. A function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is called a contra tri - continuous [4] if $f^{-1}(O)$ is tri - clopen set in $(X, \tau_1, \tau_2, \tau_3)$ for every tri - open set O in $(Y, \sigma_1, \sigma_2, \sigma_3)$.

Definition 2.9. A function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ from a tri - topological space X into a tri - topological space Y is called a tri - βg^* irresolute [9] if $f^{-1}(V)$ is tri - βg^* closed set in X for every tri - βg^* closed set V in Y .

Theorem 2.10. Every tri - open set is tri - βg^* open and every tri - closed set is tri - βg^* closed set.

3. STRONGLY Tri - βg^* CONTINUOUS FUNCTION

We introduce the following definition.

Definition 3.1. A function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is called a Strongly tri - βg^* continuous if the inverse image of every tri - βg^* clopen set in $(Y, \sigma_1, \sigma_2, \sigma_3)$ is tri - open in $(X, \tau_1, \tau_2, \tau_3)$.

Definition 3.2. A tri - topological space X is said to be tri - $\beta g^* T_{1/2}$ space if every tri - βg^* open set of X is tri - open in X .

Theorem 3.3. If a map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ from a tri - topological spaces X into a tri - topological spaces Y is strongly tri - βg^* continuous then it is tri - continuous.

Proof. Let O be a tri - open set in Y . Since every tri - open set is tri - βg^* open, O is tri - βg^* open in Y . Since f is strongly tri - βg^* continuous, $f^{-1}(O)$ is tri - open in X . Therefore f is tri - continuous. $\square$

Remark 3.4. The following examples supports that the converse of the above theorem is not true in general.

Example 3.5. let $X = Y = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}\}$, $\tau_2 = \{X, \phi, \{b\}, \{a, b\}\}$, $\tau_3 = \{X, \phi, \{b, c\}\}$ and $\sigma_1 = \sigma_2 = \sigma_3 = \{Y, \phi, \{a\}\}$.

Let $g: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $g(a) = a$, $g(b) = b$, $g(c) = c$. Clearly, g is not strongly tri - βg^* continuous.

Since $\{c\}$ is tri - βg^* open set in Y but $g^{-1}\{c\} = \{c\}$ is not tri - open set of X . clearly, g is not strongly tri - βg^* continuous.

Let $g: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $g(a) = a$, $g(b) = c$, $g(c) = b$. Since $\{b, c\}$ is tri - closed set in Y but $g^{-1}\{a\} = \{a\}$, $g^{-1}\{b\} = \{c\}$, $g^{-1}\{a, b\} = \{a, c\}$ is tri - closed subset of X . However, g is tri - continuous.

Theorem 3.6. A map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ from a tri - topological spaces X into a tri - topological spaces Y is strongly tri - βg^* continuous if and only if the inverse image of every tri - βg^* closed set in Y is tri - closed in X .

Proof. Assume that f is strongly tri - βg^* continuous. Let O be any tri - βg^* closed set in Y . Then O^c is tri - βg^* tri - open in Y . Since f is strongly tri - βg^* continuous, $f^{-1}(O^c)$ is tri - open in X . But $f^{-1}(O^c) = X / f^{-1}(O)$ and so $f^{-1}(O)$ is tri - closed in X .

Conversely, assume that the inverse image of every tri - βg^* tri - closed set in Y is tri - closed in X . Then O^c is tri - βg^* closed in Y . By assumption, $f^{-1}(O^c)$ is tri - closed in X , but $f^{-1}(O^c) = X / f^{-1}(O)$ and so $f^{-1}(O)$ is tri - open in X . Therefore, f is strongly tri - βg^* continuous. $\square$

Theorem 3.7. If a map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is strongly tri - continuous then it is strongly tri - βg^* continuous.

Proof. Assume that f is strongly tri - continuous. let O be any tri - βg^* open set in Y . Since f is strongly tri - continuous, $f^{-1}(O)$ is tri - open in X . Therefore, f is strongly tri - βg^* continuous. $\square$

Remark 3.8. The converse of the above theorem need not be true.

Example 3.9. let $X = Y = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}\}$, $\tau_2 = \{X, \phi, \{b\}, \{a, b\}\}$, $\tau_3 = \{X, \phi, \{b, c\}\}$ and $\sigma_1 = \{Y, \phi, \{a, b\}\}$, $\sigma_2 = \{Y, \phi, \{b, c\}\}$, $\sigma_3 = \{Y, \phi, \{a, c\}\}$.

Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $f(a) = a$, $f(b) = b$, $f(c) = c$.

Since $\{a, b\}, \{b, c\}, \{a, c\}$ is tri - βg^* open set in Y .

But $f^{-1}\{a, b\} = \{a, b\}$, $f^{-1}\{b, c\} = \{b, c\}$, $f^{-1}\{a, c\} = \{a, c\}$ is tri - clopen set in X . clearly, f is strongly tri - βg^* continuous. Since $\{a, b\}$ is subset of Y .

But $f^{-1}\{a, b\} = \{a, b\}$, is tri - open in X , not tri - closed in X .

Therefore f is not strongly tri - continuous.

Theorem 3.10. If a map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is strongly tri - βg^* continuous then it is tri - βg^* continuous.

Proof. Let O be an tri - open set in Y . By [8] O is tri - βg^* open in Y . Since f is strongly tri - βg^* continuous $\Rightarrow f^{-1}(O)$ is tri - open in X . By [9] $f^{-1}(O)$ is tri - βg^* open set in X . Therefore, f is tri - βg^* continuous. $\square$

Remark 3.11. The converse of the above theorem need not be true.

Example 3.12. let $X = Y = \{a, b, c\}$, $\tau_1 = \tau_2 = \tau_3 = \{X, \phi, \{b\}\}$ and $\sigma_1 = \{Y, \phi, \{a, b\}\}$, $\sigma_2 = \{Y, \phi, \{a\}\}$, $\sigma_3 = \{Y, \phi, \{a, c\}\}$.

Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Since $\{a\}, \{a, c\}, \{a, b\}$ is tri - open set in Y .

But $f^{-1}\{a\} = \{a\}$, $f^{-1}\{a, c\} = \{a, c\}$, $f^{-1}\{a, b\} = \{a, b\}$ ($X, \tau_1, \tau_2, \tau_3$) is open set of X . clearly, f is tri - βg^* continuous.

Since $\{a\}$ is tri - βg^* O ($Y, \sigma_1, \sigma_2, \sigma_3$).

But $f^{-1}\{a\} = \{a\}$, is not tri - open in X . Therefore f is not strongly tri - βg^* continuous.

Theorem 3.13. If a map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is strongly tri - βg^* continuous and a map $g: (Y, \sigma_1, \sigma_2, \sigma_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is tri - βg^* continuous then $g \circ f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is tri - continuous.

Proof. Let O be any tri - open set in Z . Since g is tri - βg^* continuous, $g^{-1}(O)$ is tri - βg^* open in Y . Since f is

strongly tri - βg^* continuous $f^{-1}(g^{-1}(O))$ is tri - open in X . But $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$. Therefore, $g \circ f$ is tri - continuous. $\square$

Theorem 3.14. If a map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is strongly tri - βg^* continuous and a map $g: (Y, \sigma_1, \sigma_2, \sigma_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is tri - βg^* irresolute, then $g \circ f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is strongly tri - βg^* continuous.

Proof. Let O be any tri - βg^* open set in Z . Since g is tri - βg^* irresolute, $g^{-1}(O)$ tri - βg^* open in Y . Also f is strongly tri - βg^* continuous $f^{-1}(g^{-1}(O))$ is tri - open in X . But $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$ is tri - open in X . Hence, $g \circ f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is tri - βg^* continuous. $\square$

Theorem 3.15. If a map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is tri - βg^* continuous and a map $g: (Y, \sigma_1, \sigma_2, \sigma_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is strongly tri - βg^* continuous, then $g \circ f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is tri - βg^* irresolute.

Proof. Let O be any tri - βg^* open set in Z . Since g is strongly tri - βg^* continuous, $g^{-1}(O)$ tri - open in Y . Also f is tri - βg^* continuous, $f^{-1}(g^{-1}(O))$ is tri - βg^* open in X . But $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$. Hence, $g \circ f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is tri - βg^* irresolute. $\square$

Theorem 3.16. Let $(X, \tau_1, \tau_2, \tau_3)$ be any tri - topological spaces and $(Y, \sigma_1, \sigma_2, \sigma_3)$ be a $T_{1/2}$ space and $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be a map. then the following are equivalent.

(i) f is strongly tri - βg^* continuous

(ii) f is tri - continuous

Proof. (1) $\Rightarrow$ (2) Let O be any tri - open set in Y . By [3] O is tri - βg^* open in Y . then $f^{-1}(O)$ is tri - open in X . Hence, f is tri - continuous. (2) $\Rightarrow$ (1) Let O be any tri - βg^* open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Since, $(Y, \sigma_1, \sigma_2, \sigma_3)$ is a $T_{1/2}$ space, O is tri - open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Since, f is tri - continuous. Then $f^{-1}(O)$ is tri - open in $(X, \tau_1, \tau_2, \tau_3)$. Hence, f is strongly tri - βg^* continuous. $\square$

Theorem 3.17. Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be a map. Both $(X, \tau_1, \tau_2, \tau_3)$ and $(Y, \sigma_1, \sigma_2, \sigma_3)$ are $T_{1/2}$ space. Then the following are equivalent.

(i) f is tri - βg^* irresolute

(ii) f is strongly tri - βg^* continuous

(iii) f is tri - continuous

(iv) f is tri - βg^* continuous

Proof. The proof is obvious. $\square$

Theorem 3.18. The composition of two strongly tri - βg^* continuous maps is strongly tri - βg^* continuous.

Proof. Let O be a tri - βg^* open set in $(Z, \eta_1, \eta_2, \eta_3)$. Since g is strongly tri - βg^* continuous, We get $g^{-1}(O)$ tri - open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. By [3] $(g^{-1}(O))$ is tri-open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. As f is also strongly tri - βg^* continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is tri - open in $(X, \tau_1, \tau_2, \tau_3)$. Hence, $(g \circ f)$ is strongly tri - βg^* continuous. $\square$

Theorem 3.19. $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ and $g: (Y, \sigma_1, \sigma_2, \sigma_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ be any two maps. then their composition $(g \circ f): (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is strongly continuous if g is strongly tri - βg^* continuous and f is tri - Continuous.

Proof. Let O be a tri - βg^* open set in $(Z, \eta_1, \eta_2, \eta_3)$. Since g is strongly tri - βg^* continuous, $g^{-1}(O)$ tri - open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Since f is tri - continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is tri - open in $(X, \tau_1, \tau_2, \tau_3)$. Hence, $(g \circ f)$ is strongly tri - βg^* continuous. $\square$

4. PERFECTLY Tri - βg^* CONTINUOUS FUNCTION

Definition 4.1. A map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is said to be Perfectly tri- βg^* continuous if the inverse image of every tri - βg^* open set in $(Y, \sigma_1, \sigma_2, \sigma_3)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$.

Theorem 4.2. If a map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ from a tri - topological spaces $(X, \tau_1, \tau_2, \tau_3)$ into a topological spaces $(Y, \sigma_1, \sigma_2, \sigma_3)$ is perfectly tri - βg^* continuous then it is strongly tri - βg^* continuous.

Proof. Assume that f is perfectly tri - βg^* continuous. Let O be any tri - βg^* open set in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Since, f is perfectly tri - βg^* continuous, $f^{-1}(O)$ is tri - open in $(X, \tau_1, \tau_2, \tau_3)$. Therefore, f is strongly tri - βg^* continuous. $\square$

Remark 4.3. The converse of the above theorem need not be true.

Example 4.4. let $X = Y = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}\}$, $\tau_2 = \{X, \phi, \{c\}\}$, $\tau_3 = \{X, \phi, \{a, c\}, \{b, c\}\}$. and $\sigma_1 = \{Y, \phi, \{c\}\}$, $\sigma_2 = \{Y, \phi, \{a, c\}\}$, $\sigma_3 = \{Y, \phi, \{b, c\}\}$.

Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $g(a) = a$, $g(b) = b$, $g(c) = c$. Since $\{c\}$, $\{a, c\}$, $\{b, c\}$ is tri - βg^* open set in Y . But $f^{-1}\{c\} = \{c\}$, $f^{-1}\{a, c\} = \{a, c\}$, $f^{-1}\{b, c\} = \{b, c\}$ is tri - open in X .

Clearly, f is strongly tri - βg^* continuous.

Since $\{a, c\}$ is tri βg^* open set in $(Y, \sigma_1, \sigma_2, \sigma_3)$. But $f^{-1}\{a, c\} = \{a, c\}$ is tri - open in X , but not tri - closed in X . Therefore f is not perfectly tri - βg^* continuous.

Theorem 4.5. If a map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ from a tri - topological spaces $(X, \tau_1, \tau_2, \tau_3)$ into a topological spaces $(Y, \sigma_1, \sigma_2, \sigma_3)$ is perfectly tri - βg^* continuous then it is perfectly tri - continuous.

Proof. Let O be an tri - open set in Y . By [8] O is an tri - βg^* open set in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Since f is perfectly tri - βg^* continuous, $f^{-1}(O)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$. Therefore, f is perfectly tri - continuous. $\square$

Remark 4.6. The converse of the above theorem need not be true.

Example 4.7. let $X = Y = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{c\}\}$, $\tau_2 = \{X, \phi, \{a, b\}\}$, $\tau_3 = \{X, \phi, \{a, c\}, \{b, c\}\}$ and $\sigma_1 = \sigma_2 = \sigma_3 = \{Y, \phi, \{c\}\}$.

Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Since $\{b\}$ is tri - open set in $(Y, \sigma_1, \sigma_2, \sigma_3)$. $f^{-1}\{c\} = \{c\}$, is both tri - open and tri - closed in X . clearly, f is perfectly tri - continuous.

Since $\{a, c\}$ is tri - βg^* open in $(Y, \sigma_1, \sigma_2, \sigma_3)$.

But $f^{-1}\{a, c\} = \{a, c\}$, is tri - open in and not tri - closed in X . Therefore f is not perfectly tri - βg^* continuous.

Theorem 4.8. A map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ from a tri - topological spaces $(X, \tau_1, \tau_2, \tau_3)$ into a tri - topological spaces $(Y, \sigma_1, \sigma_2, \sigma_3)$ is perfectly tri - βg^* continuous if and only if $f^{-1}(O)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$ for every tri - βg^* closed set O in $(Y, \sigma_1, \sigma_2, \sigma_3)$.

Proof. Let O be any tri - βg^* closed set in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Then O^c is tri - βg^* open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Since, f is perfectly tri - βg^* continuous, $f^{-1}(O^c)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$. But $f^{-1}(O^c) = X/f^{-1}(O)$ and so $f^{-1}(O)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$.

Conversely, assume that the inverse image of every tri - βg^* closed set in $(Y, \sigma_1, \sigma_2, \sigma_3)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$. Let O be any tri - βg^* open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Then (O^c) is tri - βg^* closed in $(Y, \sigma_1, \sigma_2, \sigma_3)$. By assumption $f^{-1}(O^c)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$. But $f^{-1}(O^c) = X/f^{-1}(O)$ and so $f^{-1}(O)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$. Therefore, f is perfectly tri βg^* continuous. $\square$

Theorem 4.9. Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be a map, then the following statements are true.

(i) f is strongly tri - βg^* continuous

(ii) f is perfectly tri - βg^* continuous

Proof. (1) $\Rightarrow$ (2) Let O be any $(X, \tau_1, \tau_2, \tau_3)$ open set in $(Y, \sigma_1, \sigma_2, \sigma_3)$. By hypothesis, $f^{-1}(O)$ is tri-open in $(X, \tau_1, \tau_2, \tau_3)$. Since $(X, \tau_1, \tau_2, \tau_3)$ is a discrete space, $f^{-1}(O)$ is tri - closed in $(X, \tau_1, \tau_2, \tau_3)$. $f^{-1}(O)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$. Hence, f is perfectly tri - βg^* continuous. (2) $\Rightarrow$ (1) Let O be any tri - βg^* open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Then $f^{-1}(O)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$. Hence, f is strongly tri - βg^* continuous. $\square$

Theorem 4.10. If $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ and $g: (Y, \sigma_1, \sigma_2, \sigma_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ are perfectly tri - βg^* continuous, then their composition $g \circ f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is also perfectly tri - βg^* continuous.

Proof. Let O be any tri - βg^* open set in $(Z, \eta_1, \eta_2, \eta_3)$. Since, g is perfectly tri - βg^* continuous. We get that $g^{-1}(O)$ is tri - open and tri - closed in $(Y, \sigma_1, \sigma_2, \sigma_3)$. By thm [9] $g^{-1}(O)$ tri - βg^* open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Since f is perfectly continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$. Hence, $(g \circ f)$ is perfectly tri - βg^* continuous. $\square$

Theorem 4.11. If $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ and $g: (Y, \sigma_1, \sigma_2, \sigma_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ be any two maps. Then their composition is strongly tri - βg^* continuous if g is perfectly tri - βg^* continuous and f is tri - continuous.

Proof. Let O be any tri - βg^* open set in $(Z, \eta_1, \eta_2, \eta_3)$. Then, $g^{-1}(O)$ is tri - open and tri - closed in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Since, f is tri - continuous. $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is tri - open in $(X, \tau_1, \tau_2, \tau_3)$. Hence, $(g \circ f)$ is strongly tri - βg^* continuous. $\square$

Theorem 4.12. If a map $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is perfectly tri - βg^* continuous and a map $g: (Y, \sigma_1, \sigma_2, \sigma_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is strongly tri - βg^* continuous then the composition $(g \circ f): (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is perfectly tri - βg^* continuous.

Proof. Let O be any tri - βg^* open set in $(Z, \eta_1, \eta_2, \eta_3)$. Then, $g^{-1}(O)$ is tri - open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. By thm [9] $g^{-1}(O)$ is tri - βg^* open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. By hypothesis, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$. Therefore, $(g \circ f)$ is perfectly tri - βg^* continuous. $\square$

5. TOTALLY Tri - βg^* CONTINUOUS FUNCTION

Definition 5.1. A function $(X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is called totally tri - βg^* continuous functions if the inverse image of every tri - open set of $(Y, \sigma_1, \sigma_2, \sigma_3)$ is both tri - βg^* open and tri - βg^* closed subset of $(X, \tau_1, \tau_2, \tau_3)$.

Definition 5.2. A tri - topological space X is called a tri - βg^* connected if X cannot be expressed as a disjoint union of two non - empty tri - βg^* open sets.

Definition 5.3. A tri - topological space X is called a tri - connected if X cannot be expressed as a union of two disjoint non - empty tri - open sets in X .

Example 5.4. let $X = Y = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}\}$, $\tau_2 = \{X, \phi, \{b\}, \{a, b\}\}$, $\tau_3 = \{X, \phi, \{b, c\}\}$ and $\sigma_1 = \{Y, \phi, \{c\}\}$, $\sigma_2 = \{Y, \phi, \{a, c\}\}$, $\sigma_3 = \{Y, \phi, \{b, c\}\}$.

Let $g: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $g(a) = b$, $g(b) = c$, $g(c) = a$. Therefore $g^{-1}\{c\} = \{c\}$, $g^{-1}\{a, c\} = \{b, a\}$, $g^{-1}\{b, c\} = \{a, c\}$.

Hence the inverse image of every tri - open set $(Y, \sigma_1, \sigma_2, \sigma_3)$ is both tri βg^* open and tri - βg^* closed subset of $(X, \tau_1, \tau_2, \tau_3)$.

Therefore, g is totally tri βg^* continuous.

Theorem 5.5. Every totally tri - βg^* continuous functions is tri - βg^* continuous.

Proof. Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be totally tri - βg^* continuous and O be an tri - open set of $(Y, \sigma_1, \sigma_2, \sigma_3)$. Since f is totally tri - βg^* Continuous Functions, $f^{-1}(O)$ is both tri βg^* open and tri - βg^* closed in $(X, \tau_1, \tau_2, \tau_3)$. Therefore, f is tri - βg^* continuous. $\square$

Remark 5.6. The converse of the above theorem need not be true.

Example 5.7. let $X = Y = \{a, b, c\}$, $\tau_1 = \tau_2 = \tau_3 = \{X, \phi, \{b\}\}$ and $\sigma_1 = \{Y, \phi, \{a\}\}$, $\sigma_2 = \{Y, \phi, \{b\}\}$, $\sigma_3 = \{Y, \phi, \{a, b\}, \{b, c\}\}$.

Let $g: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $g(a) = a$, $g(b) = c$, $g(c) = b$. Clearly, g is not totally tri - βg^* continuous since $g^{-1}(\{a, b\}) = \{a, c\}$ is tri - βg^* closed but not tri - βg^* open. however, g is tri - βg^* continuous.

Theorem 5.8. Every totally tri - continuous functions is totally tri - βg^* continuous.

Proof. Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be totally tri - βg^* continuous and O be an tri - open set of $(Y, \sigma_1, \sigma_2, \sigma_3)$. Since, f is totally tri - Continuous Functions, $f^{-1}(O)$ is both tri open and tri closed in $(X, \tau_1, \tau_2, \tau_3)$.

Since every tri - open set is tri - βg^* open and every tri - closed set is tri - βg^* closed. $f^{-1}(O)$ is both tri - βg^* open set and tri - βg^* closed in $(X, \tau_1, \tau_2, \tau_3)$. Therefore, f is totally tri - βg^* continuous. $\square$

Remark 5.9. The converse of the above theorem need not be true.

Example 5.10. let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{X, \phi, \{c\}\}$, $\tau_2 = \{X, \phi, \{d\}\}$, $\tau_3 = \{X, \phi, \{c, d\}\}$ and $\sigma_1 = \{Y, \phi, \{b\}\}$, $\sigma_2 = \sigma_3 = \{Y, \phi, \{b, d\}\}$.

Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $f(a) = c$, $f(b) = b$, $f(c) = a$, $f(d) = d$.

since $f^{-1}(\{b\}) = \{b\}$, $f^{-1}(\{b, d\}) = \{b, d\}$ is both $(X, \tau_1, \tau_2, \tau_3)$ open and $(X, \tau_1, \tau_2, \tau_3)$ closed in X . Clearly f is totally $(X, \tau_1, \tau_2, \tau_3)$ continuous.

But f is not totally tri - continuous as the inverse image of the tri - open set $\{b\}$ of $(Y, \sigma_1, \sigma_2, \sigma_3)$ is not a tri - clopen set in $(X, \tau_1, \tau_2, \tau_3)$, and hence tri - βg^* continuous.

Theorem 5.11. Every perfectly tri - βg^* continuous map is totally tri - βg^* continuous.

Proof. Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be a perfectly tri - βg^* continuous map. Let O be an tri - open set of $(Y, \sigma_1, \sigma_2, \sigma_3)$. Then O is tri - βg^* open in $(Y, \sigma_1, \sigma_2, \sigma_3)$. Since f is perfectly tri - βg^* continuous, $f^{-1}(O)$ is both tri - open and tri - closed in $(X, \tau_1, \tau_2, \tau_3)$, implies $f^{-1}(O)$ is both tri βg^* open and tri - βg^* closed in $(X, \tau_1, \tau_2, \tau_3)$. Therefore, f is totally tri - βg^* continuous. $\square$

Remark 5.12. The converse of the above theorem need not be true.

Example 5.13. let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{X, \phi, \{a\}\}$, $\tau_2 = \{X, \phi, \{b\}\}$, $\tau_3 = \{X, \phi, \{b, c\}\}$ and $\sigma_1 = \{Y, \phi, \{a\}\}$, $\sigma_2 = \{Y, \phi, \{a, b\}\}$, $\sigma_3 = \{Y, \phi, \{a, c\}\}$.

Let $g: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $g(a) = a$, $g(b) = b$, $g(c) = c$.

since, $g^{-1}\{a\} = \{a\}$, $g^{-1}\{a, b\} = \{a, b\}$, $g^{-1}\{a, c\} = \{a, c\}$ is both tri - βg^* open and tri - βg^* closed in X . Clearly, g is totally tri - βg^* continuous. But $g^{-1}\{a, b\} = \{a, b\}$, is tri - open in X but not tri - closed in X . Therefore, g is not perfectly tri - βg^* continuous.

Remark 5.14. The concept of totally tri - βg^* continuous and strongly tri - βg^* continuous are independent of each other.

Example 5.15. let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{X, \phi, \{a\}\}$, $\tau_2 = \{X, \phi, \{b\}\}$, $\tau_3 = \{X, \phi, \{a, b\}, \{b, c\}\}$ and $\sigma_1 = \{Y, \phi, \{a\}\}$, $\sigma_2 = \{Y, \phi, \{a, b\}\}$, $\sigma_3 = \{Y, \phi, \{a, c\}\}$.

Let $g: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $g(a) = a$, $g(b) = b$, $g(c) = c$.

since, $g^{-1}\{a\} = \{a\}$, $g^{-1}\{a, b\} = \{a, b\}$, $g^{-1}\{a, c\} = \{a, c\}$ is both tri - βg^* open and tri - βg^* closed in X . Clearly, g is totally tri - βg^* continuous.

Clearly, g is totally tri - βg^* continuous but $g^{-1}\{a, c\} = \{a, c\}$ is not tri - open in X . But $g^{-1}\{a, b\} = \{a, b\}$, is tri - open in X but not tri - closed in X . Therefore, g is not strongly tri - βg^* continuous.

Theorem 5.16. If $f: X \rightarrow Y$ is a totally tri - βg^* continuous map, and X is tri - βg^* connected, then Y is an indiscrete space.

Proof. Suppose that Y is not an indiscrete space. Let A be a non - empty tri - open subset of Y .

Since, f is totally tri - βg^* continuous map, then $f^{-1}(A)$ is a non - empty tri - βg^* clopen subset of X . Then $X = f^{-1}(A) \cup (f^{-1}(A))^c$.

Thus, X is a union of two non - empty disjoint tri - βg^* open sets which is contra-diction to the fact that X is tri - βg^* connected.

Therefore, Y must be an indiscrete space. $\square$

Theorem 5.17. If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. Then $(g \circ f): X \rightarrow Z$

(i) If f is tri - βg^* irresolute and g is totally tri - βg^* continuous the $g \circ f$ is totally tri - βg^* continuous.

(ii) If f is totally tri - βg^* continuous and g is tri - continuous then $g \circ f$ is totally tri - βg^* continuous.

Proof. (i) Let O be an tri - open set in Z . Since g is totally tri - βg^* continuous, $g^{-1}(O)$ is tri - βg^* clopen in Y . Since f is tri - βg^* irresolute, $f^{-1}(g^{-1}(O))$ is tri - βg^* open and tri - βg^* closed in X . Since, $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$. Therefore, $g \circ f$ is totally tri - βg^* continuous.

(iii) Let O be an tri - open set in Z . Since g is tri - continuous, $g^{-1}(O)$ is tri - open in Y . Since, f is totally tri - βg^* continuous, $f^{-1}(g^{-1}(O))$ is tri - βg^* clopen in X . Hence, $g \circ f$ is totally tri - βg^* continuous. $\square$

6. SLIGHTLY Tri - βg^* CONTINUOUS FUNCTIONS

Definition 6.1. A function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is called slightly tri- βg^* continuous at a point $x \in X$ if for each tri - clopen subset V of Y containing $f(x)$, there exists a tri - βg^* open subset U in X containing x such that $f(U) \subseteq V$. The function f is said to be slightly tri - βg^* continuous at each of its points.

Definition 6.2. A function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is called slightly tri- βg^* continuous if the inverse image of every tri - clopen set in Y is tri - βg^* open in X .

Definition 6.3. A function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is called contra tri - βg^* continuous functions if $f^{-1}(O)$ is tri - βg^* closed in $(X, \tau_1, \tau_2, \tau_3)$ for every tri - open set O in $(Y, \sigma_1, \sigma_2, \sigma_3)$.

Definition 6.4. A space $(X, \tau_1, \tau_2, \tau_3)$ is called a locally indiscrete space if every tri - open set of X is tri - closed in X .

Example 6.5. let $X = Y = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}\}$, $\tau_2 = \{X, \phi, \{b\}\}$, $\tau_3 = \{X, \phi, \{a, b\}, \{b, c\}\}$ and $\sigma_1 = \{Y, \phi, \{a\}\}$, $\sigma_2 = \{Y, \phi, \{a, b\}\}$, $\sigma_3 = \{Y, \phi, \{a, c\}\}$.

Let $g: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $g(a) = b$, $g(b) = a$, $g(c) = c$.

since, $g^{-1}\{a\} = \{b\}$, $g^{-1}\{a, b\} = \{a, c\}$, $g^{-1}\{a, c\} = \{b, c\}$ is both tri - βg^* open in X .

Therefore, g is slightly tri - βg^* continuous.

Theorem 6.6. For a function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$, the following state-ments are equivalent.

(i) f is slightly tri βg^* continuous.

(ii) The inverse image of every tri - clopen set O of Y is tri - βg^* open in X .

(iii) The inverse image of every tri - clopen set O of Y is tri - βg^* closed in X .

(iv) The inverse image of every tri - clopen set O of Y is tri - βg^* clopen in X .

Proof. (i) $\Rightarrow$ (ii): Follows from the proposition 6.4

(ii) $\Rightarrow$ (iii): Let O be a tri - clopen set in Y which implies O_c is tri O clopen in Y .

By (ii), $f^{-1}(O_c) = (f^{-1}(O))_c$ is tri - βg^* open in X .

(iii) $\Rightarrow$ (iv): By (ii) and (iii), $f^{-1}(O)$ is tri - βg^* clopen in X .

(iv) $\Rightarrow$ (i): Let O be a tri - clopen set in y containing $f(x)$, by (iv) $f^{-1}(O)$ is tri - βg^* clopen in X .

Take $U = f^{-1}(O)$, then $f(U) \subseteq O$.

Hence, f is slightly tri - βg^* continuous. $\square$

Theorem 6.7. Every slightly tri - continuous function is slightly tri - βg^* con-tinuous.

Proof. Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be a slightly tri - continuous function. Let O be a tri - clopen set in Y . Then, $f^{-1}(O)$ is tri - open in X . Since, every tri - open set is tri - βg^* open. Hence, f is slightly tri - βg^* continuous. $\square$

Remark 6.8. The converse of the above theorem need not be true as can be seen from the following example.

Example 6.9. let $X = Y = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}\}$, $\tau_2 = \{X, \phi, \{b\}, \{a, b\}\}$, $\tau_3 = \{X, \phi, \{b, c\}\}$ and $\sigma_1 = \{Y, \phi, \{a\}\}$, $\sigma_2 = \{Y, \phi, \{a, b\}\}$, $\sigma_3 = \{Y, \phi, \{a, c\}\}$.

Let $g: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $g(a) = b$, $g(b) = a$, $g(c) = c$.

since, $g^{-1}\{a\} = \{b\}$, $g^{-1}\{a, b\} = \{a, c\}$, $g^{-1}\{a, c\} = \{b, c\}$ is both tri - βg^* open in X . Clearly, g is slightly tri - βg^* continuous. But not slightly tri - continuous.

Since, $g^{-1}\{a\} = \{b\}$ where $\{a\}$ is tri - clopen in y but $\{b\}$ is not tri open in X .

Theorem 6.10. Every tri - βg^* continuous function is slightly tri - βg^* continu-ous.

Proof. Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be a tri - βg^* continuous function. Let O be a tri - clopen set in Y .

Then, $f^{-1}(O)$ is tri - βg^* open in X and tri - βg^* closed in X . Hence, f is slightly tri - βg^* continuous. $\square$

Remark 6.11. The converse of the above theorem need not be true as can be seen from the following example.

Example 6.12. let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{X, \phi, \{c\}\}$, $\tau_2 = \{X, \phi, \{d\}, \{a, b\}\}$, $\tau_3 = \{X, \phi, \{c, d\}\}$ and $\sigma_1 = \{Y, \phi, \{b\}, \{a, b\}\}$, $\sigma_2 = \{Y, \phi, \{b, c\}\}$, $\sigma_3 = \{Y, \phi, \{a, b, c\}\}$.

Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$.

since, $f^{-1}\{c, d\} = \{c, d\}$ is tri - βg^* open in X . The function f is slightly tri - βg^* continuous.

Let $g: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $g(a) = a$, $g(b) = d$, $g(c) = c$, $g(d) = d$.

But not tri - βg^* continuous, Since $g^{-1}\{a, b\} = \{a, b\}$ is not tri - βg^* open in X .

Theorem 6.13. Every contra tri - βg^* continuous function is slightly tri - βg^* continuous.

Proof. Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be a contra tri - βg^* continuous function. Let O be a tri - clopen set in Y . $f^{-1}(O)$ is tri - βg^* open in x as f is contra tri - βg^* continuous.

Hence, f is slightly tri - βg^* continuous. $\square$

Remark 6.14. The converse of the above theorem need not be true as can be seen from the following

example.

Example 6.15. let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{X, \phi, \{c\}\}$, $\tau_2 = \{X, \phi, \{d\}\}$, $\tau_3 = \{X, \phi, \{c, d\}\}$ and $\sigma_1 = \{Y, \phi, \{b\}\}$, $\sigma_2 = \{Y, \phi, \{a, b\}\}$, $\sigma_3 = \{Y, \phi, \{a, b, c\}\}$.

Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ be defined by $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$.

since, $f^{-1}\{b\} = \{b\}$ is tri - βg^* open in X . The function f is slightly tri - βg^* continuous.

Remark 6.16. Composition of two slightly tri - βg^* continuous need not be slightly tri - βg^* continuous as it can be seen from the following examples.

Example 6.17. let $X = Y = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{b\}\}$, $\tau_2 = \{X, \phi, \{a, b\}\}$, $\tau_3 = \{X, \phi, \{b, c\}\}$ and $\sigma_1 = \{Y, \phi, \{c\}\}$, $\sigma_2 = \{Y, \phi, \{a, c\}\}$, $\sigma_3 = \{Y, \phi, \{b, c\}\}$.

Define $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Since $f^{-1}\{c\} = \{c\}$, $f^{-1}\{a, c\} = \{b, c\}$ is open in X .

Clearly, f is slightly tri - βg^* continuous.

Define $g: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ by $g(a) = a$, $g(b) = b$, $g(c) = c$. Since $f^{-1}\{c\} = \{c\}$ is tri - βg^* open in Y .

Clearly, g is slightly tri - βg^* continuous.

But $(g \circ f): (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is not slightly tri - βg^* continuous, since $(g \circ f)^{-1}\{b\} = f^{-1}(g^{-1}\{b\}) = f^{-1}\{b\}$ is not a tri - βg^* open in $(X, \tau_1, \tau_2, \tau_3)$.

Theorem 6.18. Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. then the following properties hold:

- If f is tri - βg^* irresolute and g is slightly tri - βg^* continuous then $(g \circ f)$ is slightly tri - βg^* continuous.
- If f is tri - βg^* irresolute and g is tri - βg^* continuous then $(g \circ f)$ is slightly tri - βg^* continuous.
- If f is tri - βg^* irresolute and g is slightly tri - continuous then $(g \circ f)$ is slightly tri - βg^* continuous.
- If f is tri βg^* continuous and g is slightly tri - continuous then $(g \circ f)$ is slightly tri - βg^* continuous.
- If f is strongly tri - βg^* continuous and g is slightly tri - βg^* continuous then $(g \circ f)$ is slightly tri - βg^* continuous.
- If f is slightly tri - βg^* continuous and g is perfectly tri - βg^* continuous then $(g \circ f)$ is slightly tri - βg^* irresolute.
- If f is slightly tri - βg^* continuous and g is contra tri - βg^* continuous then $(g \circ f)$ is slightly tri - βg^* continuous.
- If f is tri - βg^* irresolute and g is contra tri - βg^* continuous then $(g \circ f)$ is slightly tri - βg^* continuous.

Proof. (i) Let O be a tri - clopen set in z . Since, g is slightly tri - βg^* continuous, $g^{-1}(O)$ is tri - βg^* open in Y . Since, f is tri - βg^* irresolute, $f^{-1}(g^{-1}(O))$ is tri - βg^* open in X . Since, $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$, $g \circ f$ is slightly tri βg^* continuous.

Let O be a tri - clopen set in z . Since, g is Tri - βg^* continuous, $g^{-1}(O)$ is tri - βg^* open in Y . Since, f is βg^* irresolute, $f^{-1}(g^{-1}(O))$ is tri - βg^* open in X . Hence, $(g \circ f) f$ is slightly tri - βg^* continuous.

Let O be a tri - clopen set in z . Since, g is slightly tri - continuous, $g^{-1}(O)$ is tri - βg^* open in Y . Since, f is tri - βg^* irresolute, $f^{-1}(g^{-1}(O))$ is tri - βg^* open in X . Hence, $(g \circ f) f$ is slightly tri βg^* continuous.

Let O be a tri - clopen set in z . Since, g is slightly Tri - βg^* continuous, $g^{-1}(O)$ is tri - βg^* open in Y . Since, f is tri - βg^* continuous, $f^{-1}(g^{-1}(O))$ is tri - βg^* open in X . Hence, $(g \circ f) f$ is slightly Tri βg^* continuous.

Let O be a Tri - clopen set in z . Since, g is slightly tri - βg^* continuous, $g^{-1}(O)$ is tri - βg^* open in Y . Since, f is strongly tri - βg^* continuous, $f^{-1}(g^{-1}(O))$ is tri - βg^* open in X . Therefore, $(g \circ f) f$ is slightly tri βg^* continuous.

Let O be a tri - open set in z . Since, g is perfectly tri - βg^* continuous, $g^{-1}(O)$ is tri - βg^* closed in Y . Since, f is strongly tri - βg^* continuous, $f^{-1}(g^{-1}(O))$ is tri - βg^* open in X . Hence, $(g \circ f) f$ is tri βg^* irresolute.

Let O be a tri - clopen set in z . Since, g is contra tri - continuous, $g^{-1}(O)$ is tri-open and tri - closed in Y . Since, f is strongly tri - βg^* continuous, $f^{-1}(g^{-1}(O))$ is βg^* open in X . Hence, $(g \circ f) f$ is slightly tri βg^* continuous.

Let O be a tri - clopen set in z . Since, g is contra tri - continuous, $g^{-1}(O)$ is tri - βg^* open and tri - βg^* closed in Y . Since, f is tri - βg^* irresolute, $f^{-1}(g^{-1}(O))$ is tri - βg^* open and tri - βg^* closed in X . Hence, $(g \circ f) f$ is slightly tri - βg^* continuous. $\square$

Theorem 6.19. If the function $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is slightly tri - βg^* continuous and $(X, \tau_1, \tau_2, \tau_3)$ is tri - βg^* $T_{1/2}$ space, then f is slightly tri - continuous.

Proof. Let O be a tri - clopen set in Y . Since, f is slightly tri - βg^* continuous, $f^{-1}(O)$ is tri - βg^* open in X . Since, X is tri - βg^* $T_{1/2}$ space, $f^{-1}(O)$ is tri - open in X . Hence, f is slightly tri - continuous. $\square$

Theorem 6.20. Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ and $g: (Y, \sigma_1, \sigma_2, \sigma_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ be functions. If f is surjective and pre tri - βg^* open and $(g \circ f): (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is slightly tri - βg^* continuous, then g is slightly tri - βg^* continuous.

Proof. Let O be a tri - clopen set in $(Z, \eta_1, \eta_2, \eta_3)$.

Since, $(g \circ f): (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is slightly tri - βg^* continuous, $f^{-1}(g^{-1}(O))$ is tri - βg^* open in X .

Since, f is surjective and Pre tri - βg^* open $f(f^{-1}(g^{-1}(O))) = g^{-1}(O)$ is tri - βg^* open in Y .

Hence, g is slightly tri - βg^* continuous. $\square$

Theorem 6.21. Let $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ and $g: (Y, \sigma_1, \sigma_2, \sigma_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ be functions.

If f is surjective and pre tri - βg^* open and tri - βg^* irresolute, then $(g \circ f): (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is slightly tri - βg^* continuous if and only if g is slightly tri - βg^* continuous.

Proof. Let O be a tri - clopen set in $(Z, \eta_1, \eta_2, \eta_3)$. Since, $(g \circ f): (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is slightly tri - βg^* continuous, $f^{-1}(g^{-1}(O))$ is tri - βg^* open in X . Since, f is surjective and Pre tri - βg^* open $f^{-1}(g^{-1}(O)) = g^{-1}(O)$ is tri - βg^* open in Y .

Hence, g is slightly tri - βg^* continuous. Conversely, Let g is slightly tri - βg^* continuous.

Let O be a tri - clopen set in (Z, η) , then $g^{-1}(O)$ is tri - βg^* open in Y .

Since, f is tri - βg^* irresolute, $f^{-1}(g^{-1}(O))$ is tri - βg^* open in X .

Hence, $(g \circ f): (X, \tau_1, \tau_2, \tau_3) \rightarrow (Z, \eta_1, \eta_2, \eta_3)$ is slightly tri - βg^* continuous. $\square$

Theorem 6.22. If f is slightly tri - βg^* continuous from a tri - βg^* connected space $(X, \tau_1, \tau_2, \tau_3)$ onto a space $(Y, \sigma_1, \sigma_2, \sigma_3)$ then Y is not a discrete space.

Proof. Suppose that Y is a discrete space. Let O be a proper non - empty tri - open subset of Y . Since, f is slightly tri - βg^* continuous, $f^{-1}(O)$ is a proper non empty tri - βg^* clopen subset of X which is contradiction to the fact that X is tri - βg^* connected. $\square$

Theorem 6.23. If $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is a slightly tri - βg^* continuous surjection and X is Tri - βg^* connected, then Y is tri - connected.

Proof. Suppose Y is not tri - connected, then there exists non - empty disjoint tri-open sets U and V such that $Y = U \cup V$. therefore, U and V are tri - clopen sets in Y .

since, f is slightly tri - βg^* continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are non - empty disjoint tri - βg^* open in X and $X = f^{-1}(U) \cup f^{-1}(V)$.

This shows that X is not tri - βg^* connected.

this is a contradiction and hence, Y is tri - connected. $\square$

Theorem 6.24. If $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is a slightly tri - βg^* continuous and $(Y, \sigma_1, \sigma_2, \sigma_3)$ is a locally indiscrete space then f is tri - βg^* continuous.

Proof. Let O be an tri - open subset of Y .

Since, $(Y, \sigma_1, \sigma_2, \sigma_3)$ is a locally indiscrete space, O is tri - closed in Y . Since, f is slightly tri - βg^* continuous, $f^{-1}(O)$ is tri - βg^* open in X . Hence, f is tri - βg^* continuous. $\square$

Theorem 6.25. If $f: (X, \tau_1, \tau_2, \tau_3) \rightarrow (Y, \sigma_1, \sigma_2, \sigma_3)$ is a slightly tri - βg^* continuous and A is an tri - open subset of X then the restriction $f|_A: (A, \tau_A) \rightarrow (Y, \sigma)$ is slightly tri - βg^* continuous.

Proof. Let V be a tri - clopen subset of Y . Then $(f|_A)^{-1}(V) = f^{-1}(V) \cap A$. Since $f^{-1}(O)$ is tri - βg^* open and A is tri - open, $(f|_A)^{-1}(V)$ is tri - βg^* open in the relative topology of A . Hence, $f|_A: (A, \tau_A) \rightarrow (Y, \sigma)$ is slightly tri - βg^* continuous. $\square$

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