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Accurate Connected Domination Number of Middle Graphs and Central Graphs of Some Families of Graphs

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Abstract
A set D be a connected dominating set if the chosen sub graph G is connected and V −D do not have a |D|, then a G's dominating set D is an accurate connected domination. An ACD number $\gamma_{ac}(G)$ represents the ACDS's minimum cardinality. This paper we find the accurate connected domination of central graph $\gamma_{ac}(C(G))$ and middle graph $\gamma_{ac}(M(G))$ for the new parameters, and precise values for some standard graphs.

Keywords: Accurate connected domination, Middle graphs, Central graphs

1. Introduction

In a G, a "subset of vertices $S \subseteq V$ is defined as a dominating set if every vertex in $V - S$ is adjacent to at least one vertex in S. The domination number, denoted by $\gamma(G)$, represents the minimum cardinality among all dominating sets of G". A concept of domination and various extensions has been extensively calculated. Much of the contemporary interest in this field was catalyzed by the seminal survey work of Cockayne and Hedetniemi (1977). Throughout this discussion, q and p represents the no. of edges and vertices respectively, while $\Delta(G)$ represents a greatest degree of any V (G). Furthermore, a vertex vV is classified as "a cut vertex if its removal increases the number of connected components in G". The diam(G) is "the maximum distance between any pair of vertices in G". Here, we showed the results of the connected dominating set for specific graph classes.

2. Literature and survey

[6] The middle graph was introduced in the paper of Yoshimura, and they worked on some classes of graphs, By adding new node to every edges of real graph and joining the newly acquired nodes if they are neighbour edges in the real graph, the middle graph is created. Kazemne jad and Moradi researched the total domination number of central graphs and Patil and Pandiya Raj explored the whole graph of central graphs and their covering numbers. Jahfar and Chithra researched a further extension on central graphs by applying some algebraic approaches, and they built some applications for finding the number of spanning trees and the Kirchhoff index in such extended graphs. The precise ACD number of central and middle graphs is the main topic of this study.

3. **PRELIMINARIES**

ACCURATE DOMINATING SET

[1] "If there is no dominating set of cardinality D in the set $V - D$, then the dominating set D of the $G = (V, E)$ is an accurate dominating set . The accurate domination number is the least cardinality of the accurate dominating set $\gamma_a(G)$".

CONNECTED DOMINATING SET

"A connected dominating set D of a G is one that forms a connected subgraph in G. since a dominating set must contain minimum one edge from each component of G, a connected dominating set exists for a G if and only if G is connected and it is denoted by $\gamma_c(G)$".Ref. Fig 1[13]

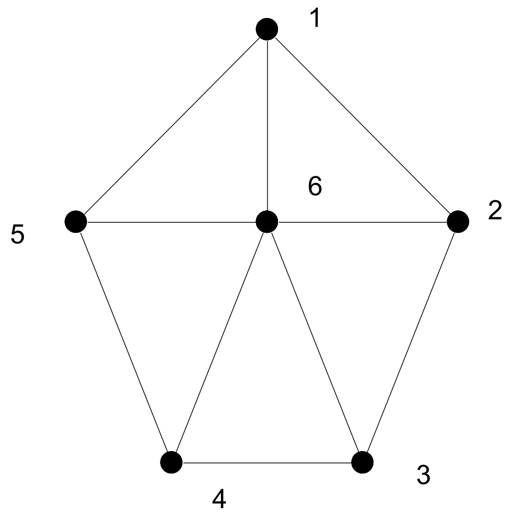

Figure 1. Wheel Graph

$\boldsymbol{\gamma(G) = 1, \gamma_a(G) = 1\ \gamma_c(G) = 1}$

CENTRAL GRAPHS

"The central graph C(G) is created by connecting all of G′s non-adjacent vertices in C(G) and subdividing each E(G) uniquely."

MIDDLE GRAPHS

"A middle graph of connected graph represented by M(G), have a vertex set of V ∪ E, where two nodes are neighbour by if, they are neighbour edges or incident with it."

ACCURATE CONNECTED DOMINATING SET

"The connected dominating set D of G = (V,E) is an ACDS, if the generated sub graph D is connected and V −D does not have a dominating set of cardinality D. The ACD number $\gamma_{ac}(G)$ is the least cardinality of the accurate connected dominating set." Ref Fig. 2

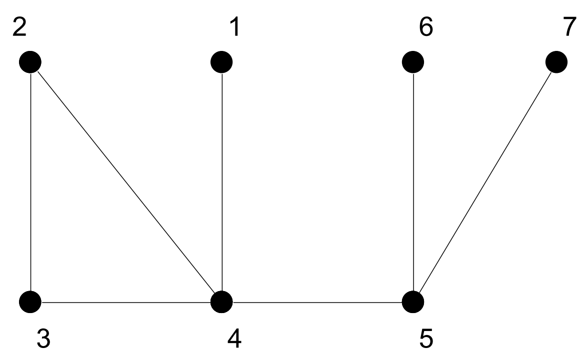

Figure 2

$\mathbf{D = \{4,5\}},\ \ \mathbf{V - D = \{1,2,3,6,7\}},\ \ \boldsymbol{\gamma_{ac}}\mathbf{(G) = \{4,5\} = 2}$

4 . ACCURATE CONNECTED DOMINATION NUMBER OF MIDDLE GRAPHS

THEOREM 4.1

For every path graph of a middle graphs $\boldsymbol{\gamma_{ac}}\,(M(P_n)) = n - 1$ for $n \geq 3$

Proof :

Consider $v_1, v_2, v_3, \dots, v_n$ be the nodes of the path P_n and let $e_1, e_2, e_3, \dots, e_{n-1}$be the newly included vertices corresponding to the edges of P_n . Let $V(P_n)$ U $E(P_n)$ be the vertex of (P_n) . Then $D = \{e_1, e_2, e_3, \dots, e_{n-1}\}$ be the accurate connected dominating set of graph G and the remaining nodes $v_1, v_2, v_3, \dots, v_n$ has no connected dominating set of cardinality.

Hence any path middle graph 's

$$V(M(P_n)) = 2n - 1$$
$$E\,(M(P_n)) = 3n - 4$$

So $n = 3$,

$$V[M(P_3)] = 5$$
$$E\,[M(P_3)] = 5$$

Therefore

$$\gamma_{ac}\,(M(p_n)) = n - 1$$

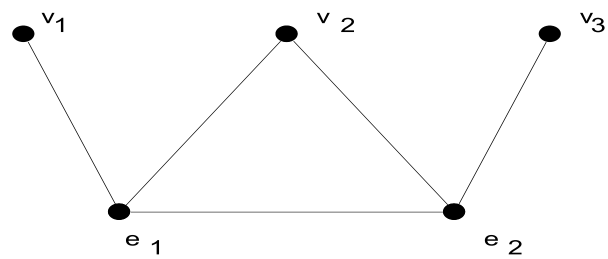

Figure 3 Middle graph of a Pathgraph
$$\boldsymbol{\gamma_{ac}\,(M(p_3)) = 2}$$

THEOREM 4.2
For any cycle graph of middle graph $\gamma_{ac}\,(M(C_n)) = n - 1$ for $n \geq 3$
Proof :
Consider $V(C_n) = \{v_1, v_2, v_3, \ldots, v_n\}$ and $E(C_n) = \{e_1, e_2, e_3, \ldots, e_n\}$ where $e_i = v_i v_{i+1}$ For $(1 \leq i \leq n-1)$ and $e_n = v_n v_1$. Let $V(C_n)\,U\,E(C_n)$ be vertex of $M(C_n)$ and let $D = \{e_1, e_2, e_3, \ldots, e_{n-1}\}$ be the accurate connected dominating set and the left out other vertices does not satisfy the condition of accurate connected dominating set,
Hence for any cycle middle graph's

$$V(M(C_n)) = 2n$$
$$E\,(M(C_n) = 3n$$

So for $n = 5$

$$V[M(C_5)] = 10$$
$$E\,[M(C_5)] = 15$$

Therefore

$$\gamma_{ac}\,(M(C_n)) = n - 1$$

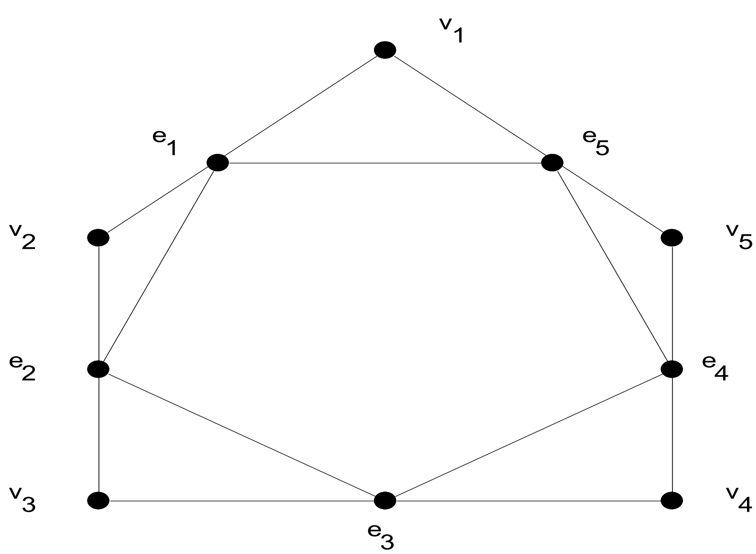

Figure 4 Middle graph of a Cycle graph

$$\boldsymbol{\gamma_{ac}\,(M(C_5))\ =\ 4}$$

THEOREM 4.3

The middle graph of a wheel graph $w_{1,n}$ accurate connected dominating set is $\gamma_{ac}\,(M(w_{1,n}))\ =\ n\ \ for\ n\ \geq 3$

Proof :

The nodes of $w_{1,n}$ be $v_1, v_2, v_3, \dots, v_{n+1}$ and $e_i = (v_{i+1}, v_{i+2})$ for $i = 1,2,3, e_i = \{v_1, v_{i-3}\}$ for $i =$ 5,6,7,8 $e_i = (v_{i-2}, v_{i+1})$ for $i = 4$ be the edges of $w_{1,n}$. Here v_1 be the centre vertex and the other vertices $v_2, v_3, \dots, v_n$ on the rim and for $n = 4$, consider $D = \{e_5, e_6, e_7, e_8\}$ be the accurate connected dominating set and the remaining vertices does not satisfy the condition of the accurate connected dominating set.

Hence for any wheel middle graph 's

$$V(M(w_{1,n}))\ =\ 3n + 1$$
$$E(M(w_{1,n}))\ =\ 6n$$
$$\gamma_{ac}\left(M\left(w_{1,n}\right)\right) = n \qquad for\ n\ \geq 3$$

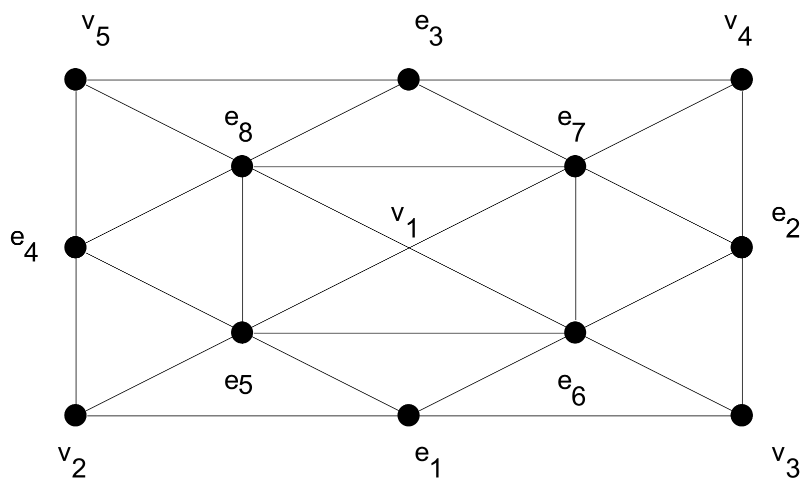

Figure 5 Middle graph of a Wheel graph

$$\boldsymbol{\gamma_{ac}\,(M(w_{1,4}))\ =\ 4}$$

THEOREM 4.4

The ACD of a middle graph of a sunlet graph $\gamma_{ac}\,(M(S_n))\ =\ 2n\quad for\ n\ \geq 4$

Proof : Consider $\{v_1, v_2, v_3, \dots, v_{2n}\}$ be the nodes and $\{e_1, e_2, e_3, \dots, e_{2n}\}$ be the edges of the sunlet graph. By the definition of sunlet graph each edges are subdivided by the vertices, therefore the nodes set $V(M(S_n)) = \{v_1, v_2, v_3, \dots, v_{2n}\}\ U\ \{e_1, e_2, e_3 \dots, e_{2n}\}$. Let $D = \{v_8, e_1, e_2 \dots, e_{2n}\}$ be the accurate connected dominating set and since each and every vertex of D dominates maximum vertices and other vertices does not connected so it does not satisfies the condition of accurate connected dominating set. Ref Fig 6

$$V(M(S_n))\ =\ 4n$$
$$E(M(S_n))\ =\ 7n$$

Hence for any sunlet middle graph is

$$\gamma_{ac}\,(M(S_n))\ =\ 9$$

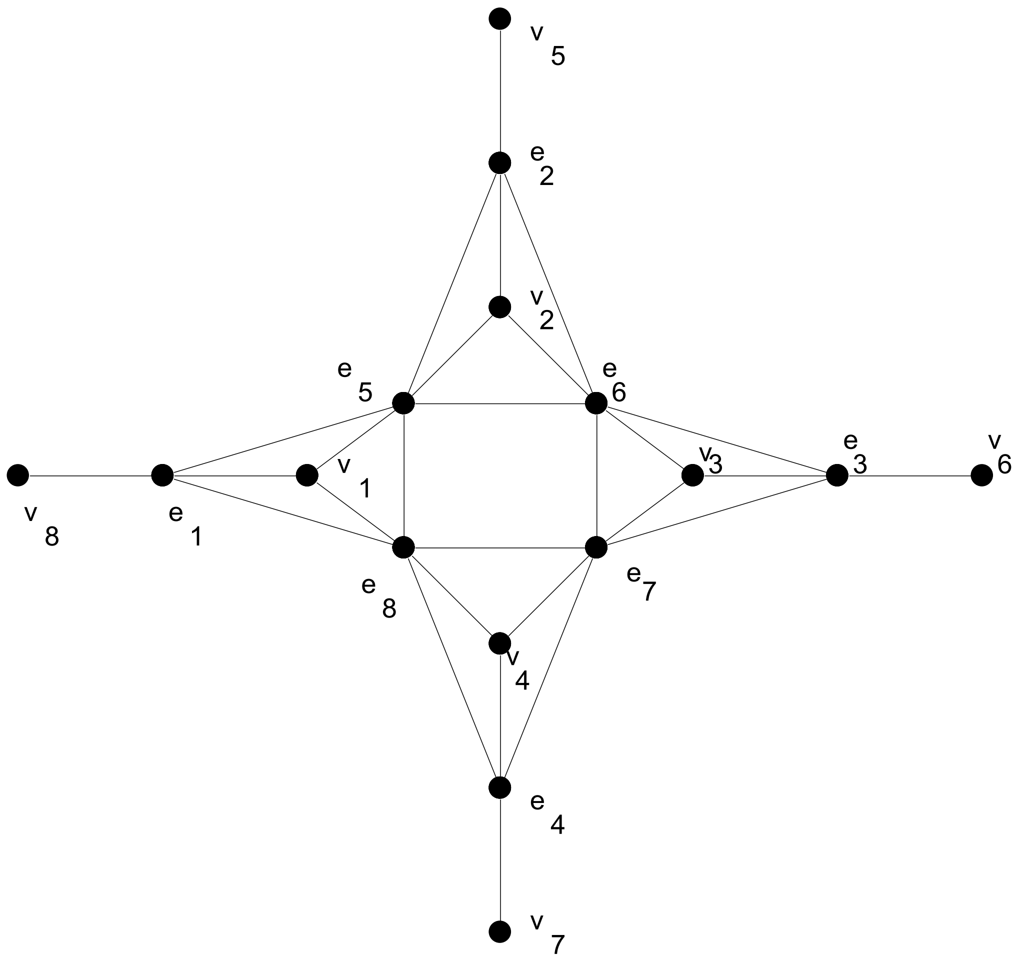

Figure 6 Middle graph of 4-Sunlet graph

$$\boldsymbol{\gamma_{ac}\ (M(S_4))\ =\ 9}$$

THEOREM 4.5

In the M(G) of star graph the ACD number is $\gamma_{ac}\ (M(K_{1,n}))\ =\ n$

Proof :

"Let $V\ (K_{1,n}\ )\ = \{v, v_1, v_2, v_3, \dots, v_n\ \}$and $E(G)\ =\ \{e_1, e_2, e_3, \dots, e_n\}$ and v is the centre of vertex. By middle graph $V(M(K_{1,n}))\ =\ \{v\}U\{e_i\ /\ 1\ \leq\ i\ \leq\ n\ \}U\ \{v_i\ /1\ \leq\ i\ \leq\ n\ \}$ in which the vertices $e_1, e_2, \dots, e_n$, be the accurate connected dominating set and the other remaining vertices is not connected and does not dominates,"

$$V(M(K_{1,n}))\ =\ 2n+1$$

$$E(M(K_{1,n}))\ =\ 3n$$

Hence for any star middle graph is

$$\boldsymbol{\gamma_{ac}\ (M(K_{1,n}))\ =\ n}$$

5. Accurate connected domination(ACD) number of central graphs

THEOREM 5.1

Any central graph of cycle C_n, $\gamma_{ac}\ (C(C_n))\ =\ 2n-3\quad for\ n\ \geq\ 4$

Proof:

Consider $v_n, v_{n-1}, \dots, v_1$ be the nodes of c_n and $u_n, u_{n-1}, \dots, u_1$ be the Middle vertices to the respective edges $v_1v_2, v_2v_3, v_3v_4, \dots, v_1v_n$ where $1\ \leq\ i\ \leq\ n$ of C_n to get $C\ (C_n\ )$. Then

$$|E(C(C_n)\ |\ = 3n\ and\ |V(C(C_n\ )\ )\ |\ =\ 2n$$

For $n\ =\ 5$. Let $D\ =\ \{v_1, u_1, v_2\ , u_2, v_3, u_3, v_4\ \}$ be accurate connected dominating set and the other $V-D$ vertices does not dominate, so the accurate connected dominating set of $C(C_n)$ is Ref Fig 7

$$\gamma_{ac}\ (C(C_n))\ =\ 2n-3$$

U5

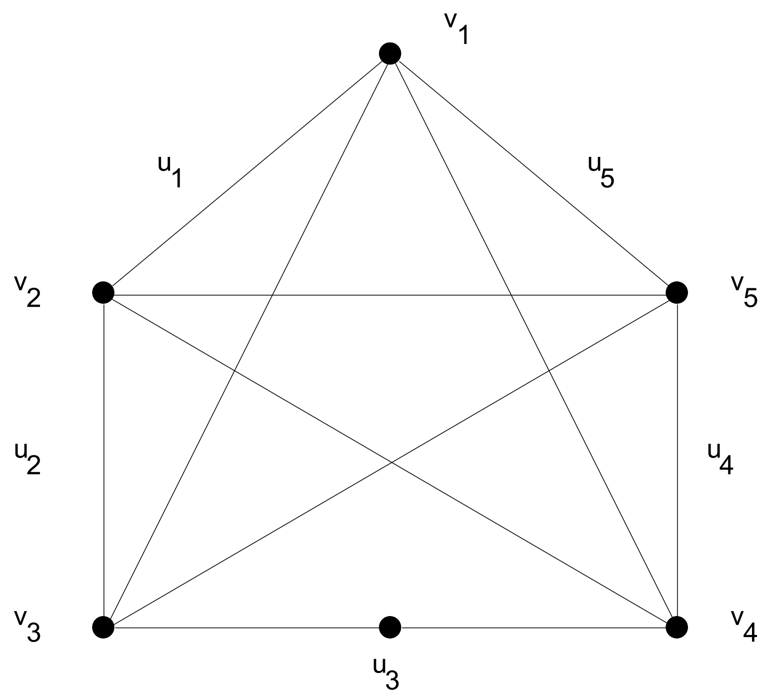

Figure 7 Central graph of a cycle graph
$\boldsymbol{\gamma_{ac}\ (C(C_5\ ))\ =\ 7}$

THEOREM 5.2
Any central graph of P_n , $\gamma_{ac}\ (C(P_n))\ =\ 2n-3$
Proof:
Consider P_n be path of length n with the nodes and u_i be the vertex of subdivision of edges $v_i v_{i+1}$ $(n \geq i \geq 1)$. In $C(P_n)$ the vertex v_i is adjacent with all the vertices except the nodes v_{i+1} and v_{i-1} for $1\ \leq\ i\ \leq\ n-1$.
For $n\ = 5$ consider $D\ =\ \{u_1, v_2\ , u_2, v_3, u_3, v_4, u_4\ \}$, be accurate connected dominating set and $V-D$ does not dominate So accurate connected dominating set of $C(P_n)$ is Ref Fig 8
$\gamma_{ac}\ (C(P_n))\ =\ 2n-3$

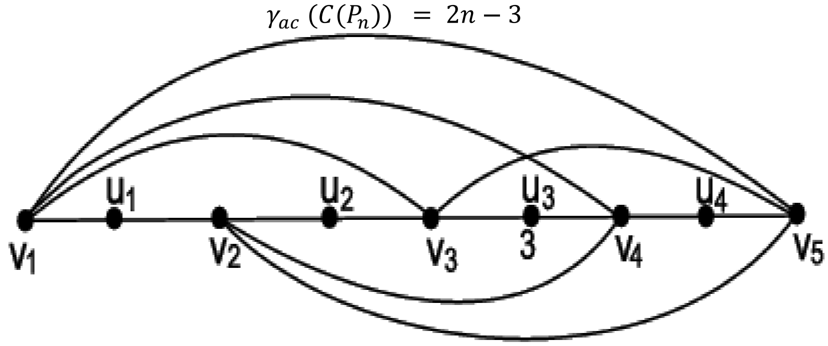

Figure 8 Central graph of a path graph
$\boldsymbol{\gamma_{ac}\ (C(P_5))\ =\ 7}$

Theorem 5.3
Any central graph of wheel graph $w_{1,n}$, $\gamma_{ac}\ (C(w_{1,n}\ ))\ =\ 3\ for\ n\ \geq\ 3$
Proof :
The wheel graph $w_{1,n}$ consists of the following nodes $v_0, v_1, v_2, \ldots, v_n$. Let v_0 be the centre of $w_{1,n}$. To obtain $C(w_{1,n}\ )$ add $u_1, u_2, \ldots, u_n$ as the included nodes corresponding to edges $w_{1,n}$. The central graph's vertex and edges sets of wheel graphs are $2n\ +\ 1$ and $2n\ + nc_2$. Consider $D\ =\ \{v_0\ , u_1, v_4\ \}$ be the accurate connected dominating set and remaining $V-D$ does not satisfy the condition of acd Ref Fig 9. Therefore

$$\gamma_{ac}\left(C(w_{1,n})\right) = 3$$

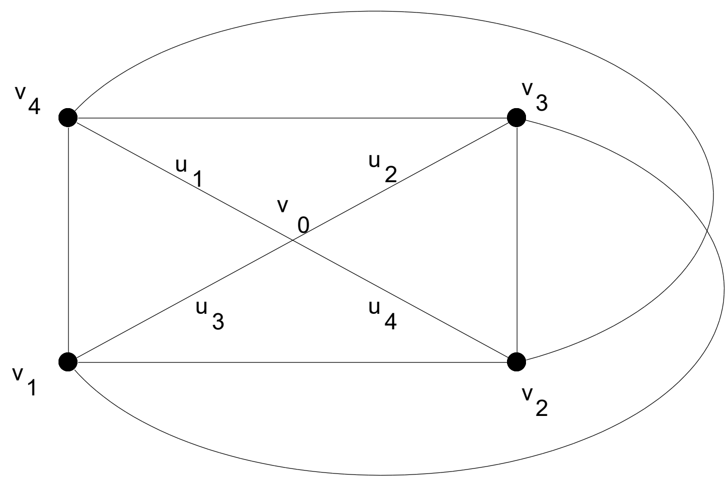

Figure 9 Central graph of a Wheel graph

$$\boldsymbol{\gamma_{ac}\left(C(w_{1,4})\right) = 3}$$

Theorem 5.4

For any central graph of star graph $k_{1,n}$, $\gamma_{ac}\left(C(k_{1,n})\right) = 3$

Proof :

The central graph $C(k_{1,n})$ is considered by subdividing each of v_0v_i of $k_{1,n}$ exactly once by the nodes v_i , $n \geq i \geq 1$ in $C(k_{1,n})$ and connecting non neighbour nodes. Let $u_n, u_{n-1}, \dots, u_1$ be the middle vertices of v_0v_i corresponding to each vertices, Then

$V(C(K_{1,n})) = 2n + 1$ and $E(C(K_{1,n})) = 2n + nc_2$.

The set $D = \{v_0, v_i, u_i\}$ be accurate connected dominating set and $V - D$ does not have dominating set of cardinality D.so

$$\gamma_{ac}\left(C(k_{1,n})\right) = 3$$

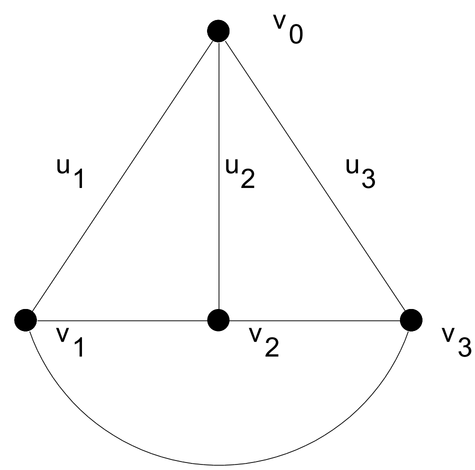

Figure 10 Central graph of a star graph

$$\boldsymbol{\gamma_{ac}\left(C(k_{1\,3})\right) = 3}$$

THEOREM : 5.5

For any central graph of pan graph $\gamma_{ac}\left(L(n,1)\right) = n + 3$

Proof :

From the definition of pan graph we observe that pan graph is acquired by connecting C_n to a singleton graph k_1. Let $\{v_1, v_2, v_3, \dots, v_n\}$ be the nodes of C_n and $\{u_1, u_2, u_3, \dots, u_n\}$ are the edges of the cycle graph C_n.For L(3,1) consider $D = \{v_1, u_1, v_2, u_2, v_3 u_5\}$ be the accurate connected dominating set and the remaining $V - D$ vertices does not dominates Ref Fig 11. so

$$\gamma_{ac}\left(L(n,1)\right) = n + 3$$

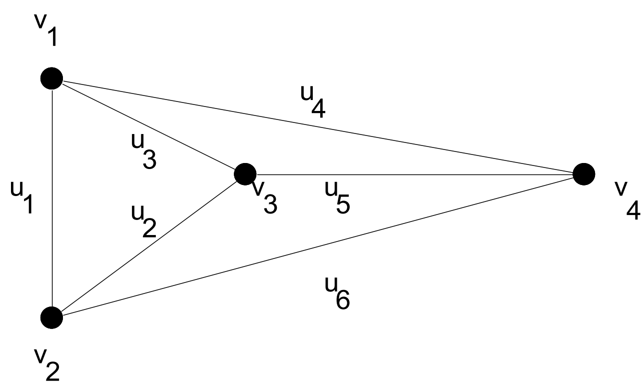

Figure 11 Central graph of a Pan graph

$$\boldsymbol{\gamma_{ac}\left(L(3,1)\right) = 6}$$

6. Conclusion

We were establish the accurate connected domination in central and middle graphs. We found the new parameter value for some families of graph and explained with examples. In future we are going find the algorithm for this new parameter and going to apply in computer science, networking etc.

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