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Identification and Quantization of Multiple Damages on 1-Dimensional Beam by Minimizing Wasserstein Distance with Sensitive Modal Amplitude Data

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Abstract

A novel Predictor-Critic Artificial Neural Network architecture is presented here which minimizes Wasserstein Distance simultaneously with Jensen Shannon Divergence, Mean Squared Error and Mean Absolute Error as regression losses. Modal Amplitude data which is input to Artificial Neural Network, has been made sensitive by elastically striding strip of tiny mass. Damage amount is converted into Probability Mass Function. Proposed Artificial Neural Network not only predicts location of multiple damages but also predicts their amount with great accuracy. The present methodology is verified on a 1-D beam with four different boundary conditions- cantilever, simply supported, overhanging and propped cantilever. Baseline Modal Amplitude Data is not required for the proposed approach.

Keywords: Damage Localization, Damage Quantification, Wasserstein Distance, Jensen-Shannon Divergence, Artificial Neural Network Regression, Structural Health Monitoring

1. Introduction

Monitoring structural damage is extremely important for sustaining and preserving the service life of mechanical, aerospace and civil structures.Condition Based Monitoring Systems (CBMS) give online information of the health, serviceability, integrity and safety of structures. Damage may occur and propagate on engineering structures due to different environmental and human-induced factors. Visual inspections have its own limitation of Structural Health Monitoring (SHM). Various Non-Destructive Testing (NDT) have been developed in recent years like Laser Testing Methods, Infrared Thermography, Acoustic Emissions, Radiographic Testing, Digital Image Correlation, Liquid Penetrant Testing, Ultrasonic Testing, Magnetic Particle Testing, etc. Recent development in Artificial Intelligence (AI) provided better tools for SHM.

Structural damage is time dependent problem where the dimensions and material properties reach to an unacceptable limit. Damages which are not detectable, chronically progress and accurate CBMS is required to identify, localize, and quantify the damage. Fine granular damages caused by corrosion etc. are undetectable by traditional techniques. Fine granular damages propagate chronically and reduce the integrity and service life of mechanical, aerospace and civil engineering structures. This research work proposes novel Predictor-Critic Artificial Neural Network (ANN) (Fig. 4 and Fig. 5) to identify, localize, and quantify multiple fine granular damages that may be caused by corrosion, among other causes.

During last decade different Machine Learning (ML) and Deep Learning (DL) tools, parametric and non-parametric, have been developed as NDT. This research addresses Free Natural Vibration bases method which can identify, localize and quantify multiple damages in the range of 0.1 mm to 0.3 mm on small 1-D beams with ANN. Few Modal Amplitude values, which are input to ANN, for such a small range of damages are ambiguous from the perspective that other combination of multiple damages may produce almost similar values. Such a challenging SHM with small data of Free Vibration based Modal Amplitudes is proposed in this research work.

Stages in CBMS [1] are Identification, Localization, Quantification and Prognosis of damages. Proposed methodology addresses first three stages of CBMS. Support Vector Machine (SVM) with kernel trick has been used to identify the multiple damages and Predictor-Critic based ANN regression has been used to localize and quantify the multiple damages on 1-D beam.

Dems et al. [2] proposed; since frequency shift measurements offer a global assessment of structural damage, they often exhibit limited sensitivity to damage magnitude, particularly in the case of localized defects such as cracks or holes. To enhance sensitivity, additional parameters are introduced into the system, such as concentrated elastic or rigid supports, masses attached either elastically or rigidly, modified boundary conditions, or applied prestress. The findings ofDems et al. [2] are further verified by results of the current research work along with Appendix A.

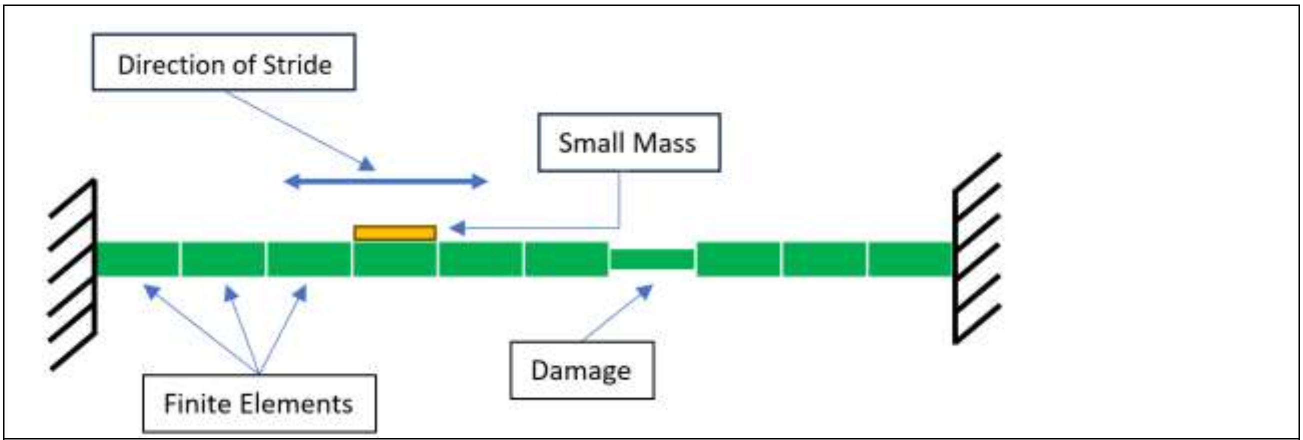


Fig. 1. Simply Supported beam with 10 finite elements and striding mass

Present work is based on finding of Dems et al. [2], but instead of using Frequencies current research work uses few modal amplitude values because it has global span. Small mass is stridden, elastically, over the beam as shown in Fig. 1. With each stride the modal amplitude data is calculated and after preprocessing it is input to ANN. The proposed ANN architecture localizes and quantifies the multiple damages better than the cases when there is no stride of mass over the beam. The current research verifies the idea of Dems et al. [2] that striding of mass makes the modal amplitude data sensitive. Modal amplitude data is calculated by Free Vibration Dynamic Finite Element Analysis (FEA).

SHM is a multidisciplinary research field that involves diverse approaches, ranging from frequency shifts compared to baseline values, wavelet transforms, changes in mode shape curvature, modifications in frequency response functions, advanced computational methods like genetic algorithms and fuzzy logic.

Variations in baseline natural frequencies serve as a clear indicator of structural damage in engineering systems. In a study of Adams et al. [3], damage was assumed to occur at an unknown axial location along a prismatic bar, with the associated reduction in stiffness. The authors employed baseline frequencies to theoretically estimate the direct receptance on both sides of the damage. By analysing the differences between the undamaged baseline and damaged natural frequencies in two vibration modes, they plotted the sum of direct receptance on either side of the suspected damage. The point where these plots intersect indicates the damage location. Chondros et al. [4] analytically established a relationship between variations in the natural frequencies of a cantilever beam and the crack depth at the weld, assuming identical Young's modulus for both the beam and the weld material. An empirical correlation was derived linking crack depth to multiple natural frequencies, enabling crack detection with an accuracy of approximately 10%.

Khiem and Lien [5] expanded on this by investigating the presence of multiple cracks in a cantilever beam. Using the Dynamic Stiffness Matrix method, they formulated the problem as a constrained convex optimization task. Their findings showed that incorporating a greater number of natural frequency measurements significantly enhances the accuracy of identifying multiple damage locations

The Wavelet Transform is highly effective for structural damage identification due to its ability to detect subtle changes in deflection profiles. Shahsavari et al. [6] applied the Continuous Wavelet Transform (CWT) to both simply supported and fixed beams for the purpose of damage detection and localization. To improve the accuracy of wavelet coefficient estimation and reduce the influence of experimental noise, they integrated Principal Component Analysis (PCA) into their approach. The resulting algorithm demonstrated strong statistical reliability in identifying damage at multiple locations.

Modal curvature shapes are highly responsive to both the location and severity of structural damage. Wahabet al. [7] introduced a technique that consolidates the variations in mode shape curvatures into a single metric to detect and pinpoint damage. This method was validated through simulations on beams with different boundary conditions and successfully applied to real-world civil engineering structures. Likewise, Pandey et al. [8] highlighted that changes in modal curvature become more pronounced as damage increases. Using Finite Element Analysis (FEA), they studied cantilever and simply supported beams to compute modal amplitude values. The deviation between the baseline and current mode shape curvatures were utilized to identify the location of damage.

The Frequency Response Function (FRF) is defined as the ratio of the Fourier Transform of a system's time-domain response to the Fourier Transform of its input signal. In their study, Dackermann et al. [9] examined the differences between the baseline and current FRFs of a two-storey framed structure. To enhance the accuracy of the results, they applied PCA to filter out noise from the FRF data. Subsequently, an ANN was employed to detect joint damage based on the processed FRF data. Similarly, Bandara et al. [10] used PCA to reduce the FRF data obtained from FEA of a two-storey frame structure. They calculated a damage index from the FRF, which was then fed into an ANN to map the location and severity of the structural damage.

Genetic Algorithms (GAs) are a widely used meta-heuristic optimization technique that addresses complex optimization problems, particularly in large, nonlinear search spaces by mimicking the process of natural selection. In the study conducted by Alves et al. [11], GA was used in combination with various damage indicators, such as natural frequencies, vibration mode shapes, modal curvatures, and frequency response functions, to detect, locate, and quantify damage in structures. These indicators offer valuable insights into the health of a structure, and GAs optimize the process of identifying and assessing damage based on these metrics.

Fuzzy logic is a logical system that extends traditional multivalued logic. In the study by Allali et al. [12], a fuzzy set was developed to assess building damage after an earthquake, categorizing damage into five levels: No damage, Slight damage, Moderate damage, Severe damage, and Collapse. These categories represent different degrees of damage, allowing for more nuanced assessments compared to binary classifications like "damaged" or "not damaged." To optimize the accuracy of these assessments, GAs were used to fine-tune the weights of the rules governing the fuzzy set. The results of their study showed that this approach achieved a high level of performance, with 90% global accordance, indicating the system's reliability for post-earthquake damage evaluation.

The experimental setups used in the reviewed studies differ significantly. Some works, such as those by Chondroset al. [4], Shahsavari et al. [6], and Allali et al. [12], involve direct measurements on real engineering structures. Others, like Adams et al. [3], focus on validating methodologies through testing on actual structures. A third category combines FEA with experimental testing on physical models, as demonstrated by Wahabet al. [7], Dackermann et al. [9], and Alves et al. [11]. Finally, certain studiessuch as Khiem et al. [5], Pandey et al. [8], and Bandara et al. [10] rely solely on FEA to verify their proposed methods.

This paper presents a novel methodology that integrates Dynamic FEA of one-dimensional beams with ANNs. As universal function approximators, ANNs are well-suited to capture complex, non-linear relationships between input parameters and output targets. Prior research, such as the works of Pawar et al. [13], Hakim et al. [14], Gordan et al. [15], Jeyasehar[16], and Rosales et al. [17], had demonstrated the effectiveness of such models in analysing beam structures. These studies utilized input features like natural frequencies and modal amplitude values to achieve objectives such as damage detection, localization, and quantification.

Pawar et al. [13] performed Dynamic FEA on a fixed-fixed beam, discretized into 20 elements, with a total length of 600 mm and a cross-sectional area of 240 mm^2. To identify structural damage, they employed a three-layer ANN. The network was trained using Fourier coefficients obtained from the spatial Fourier series expansion of damaged mode shapes. The ANN effectively predicted both the location and extent of damage. Their proposed approach was validated through dynamic FEAsimulations, demonstrating the capability of combining FEA and ANN for damage detection in one-dimensional beam structures.

Hakim et al. [14] proposed a hybrid approach that integrates FEA with ANN to detect damage in an I-section beam. The beam was modelled using eight-node linear 3D brick elements, and modal analysis was conducted to obtain the necessary input data for the ANN. The ANN featured fifteen input neurons, one representing the natural frequency and fourteen representing modal amplitude values. The output layer consisted of three neurons: two for predicting the locations of potential damage and one for estimating the severity of the damage. The I-section beam used in the simulation had the following dimensions. Length 3200 mm, Flange width 75 mm, Section depth 150 mm, Flange thickness 7 mm, Width thickness 5 mm.

Gordan et al. [15] introduced a hybrid approach combining the Imperialist Competitive Algorithm (ICA) with an ANN to detect structural damage in a system composed of three steel I-beams connected to a concrete slab, measuring 3200 mm in length. The investigation included 25 distinct damage scenarios, each defined by a constant width of 5 mm and depths varying from 3 mm to 75 mm in 3 mm increments. FEA was conducted using ABAQUS software. The steel beams were modelled with 4-node shell elements (S4R), while the concrete deck was represented using 8-node linear brick elements (C3D8R). The ANN architecture featured 15 input neurons, one corresponding to the natural frequency and the remaining 14 to mode shape data, along with a single output neuron to predict damage severity. Performance evaluation was based on the Mean Absolute Error, with results indicating that the hybrid ICA-ANN model yielded more accurate predictions than the standalone ANN model.

Jeyaseharet al. [16] considered prestressedconcrete beams of length 4860 mm with a rectangular cross-section of 125 x 250 mm under dynamic response to assess only the severity of damage. Four beams of the same size and same boundary condition have been considered with severity of damage 33.33%, 50%, 66.67% and 83.33%. ANN established non-linear relationship between degree of damage (which is output)and inputs like natural frequency, deflection, crack width, first crack load, ultimate load.

Rosales et al. [17] studied a cantilever beam with a length of 100 cm and a rectangular cross-section, where the height was 5 cm and the width was 1 cm. They considered a minimum crack depth of 0.01 m (1 cm) in their analysis. The first three natural frequencies of the beam were used as input data for the ANN. However, the testing error of the ANN was relatively high when both the location and extent of damage were predicted simultaneously. To improve their results, the location of the damage was determined using the Power Series Technique, while the amount of damage was predicted by the ANN.

An important aspect not addressed in Refs. Pawar et al. [13]; Hakim et al. [14]; Gordan et al. [15]; Jeyaseharet al. [16]; Rosales et al. [17] is that, although localized damage significantly affects local stress and strain fields, global dynamic characteristics such as natural frequencies and modal amplitudes, being indicators of overall system energy, exhibit low sensitivity to minor, localized stiffness degradations. To overcome this limitation, Dems et al. [2] proposed an analytical approach incorporating auxiliary parameters, including concentrated elastic or rigid supports and added masses, to enhance the detectability of a single localized damage.

The approach presented by Dems et al. [2] exhibits several limitations which are addressed in the current study. First, their method considers only a single damage scenario, whereas the current research work extends the analysis to multiple damages. Second, while their focus is limited to damage localization, this study addresses both localization and quantification. Third, Dems et al. [2] utilize natural frequencies from free vibration to identify damage, whereas this work employs free vibration modal amplitudes which have global span over beam, which provide a more comprehensive structural representation and enhance localization accuracy. Finally, their methodology relies on an analytical approach combined with FEA, while the current study adopts a more advanced strategy based on ANNs with a quadruple loss function.

Following are research gaps identified in literature review. These research gaps have been successfully addressed in this research paper.

1. This study employs Modal Amplitude data to detect multiple fine-scale damages ranging from 0.1 mm to 0.3 mm, smaller than those addressed in prior investigations Refs. Pawar et al. [13]; Hakim et al. [14]; Gordan et al. [15]; Jeyaseharet al. [16]; Rosales et al. [17]
2. The beam dimensions investigated in this study are smaller than those reported in all prior studies, including Pawar et al. [13], Hakim et al. [14], Gordan et al. [15], Jeyaseharet al. [16], and Rosales et al. [17].
3. None of the papers referred have addressed the problem of simultaneous identification, localization, and quantification of four damages together. This study aims to fill this gap by introducing a novel Predictor-Critic ANN based method for the simultaneous identification, localization, and quantification of four damages together.
4. A further limitation observed in the existing literature Pawar et al. [13]; Hakim et al. [14]; Gordan et al. [15]; Jeyasehar et al. [16]; Rosales et al. [17] is the predominant reliance on conventional loss functions such as Mean Squared Error (MSE) or Mean Absolute Error (MAE) in training ANNs. In contrast, the present study integrates advanced statistical measures like Wasserstein Distance and Jensen-Shannon Divergence, alongside MSE and MAE, thereby enhancing the network's sensitivity to detect fine-scale damage in the range of 0.1 mm to 0.3 mm. This level of precision is particularly valuable for structural elements with smaller cross-sectional dimensions, where such damage is often imperceptible to visual inspection.
5. A final limitation addressed in this study is that, unlike previous works, such as those by Pawar et al. [13], Hakim et al. [14], Jeyaseharet al. [16], and Rosales et al. [17], which validated their methodologies using a single boundary condition, this paper evaluates the proposed approach using four distinct beam configurations with varying boundary conditions (see Fig. 2).

2. Problem Description

The current paper considers 1-D IsotropicBeam, the material considered for the beam is steel, having material properties asModulus of elasticity E = 200x10^9 Pa, Density ρ = 7840 kg/m^3(Ref. [18]). Physical properties ofbeam as height h=1 cm, width b=1.5 cm, length l=15 cm. The beam has a rectangular cross-section, area A=b$\times$ h, moment of inertia $I = \frac{b\times h^3}{12}$.

1-D beam Finite Elementshave two nodes and each node has two Degrees of Freedom (dof).Following arethe elementary stiffness $'k'$ and mass $'m'$ matrix of beam considered (Ref. [18]).

$$k = \frac{EI}{l^3} \times \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \qquad m = \frac{\rho A l}{420} \times \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

Numerical experiments have been done on four different boundary conditions of 1-D beams which is shown in Fig. 2.

a. Cantilever Beam
b. Simply Supported Beam
c. Overhanging Beam
d. Propped Cantilever Beam

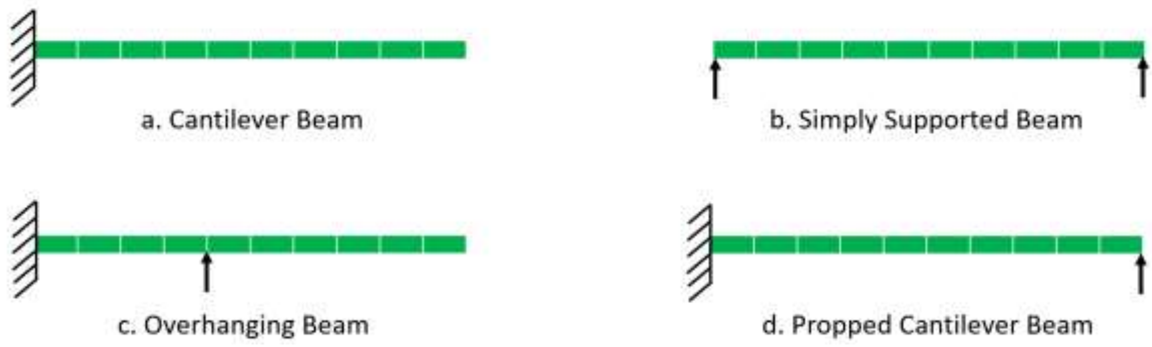


Fig. 2. Boundary conditions of 1-D beams under consideration

Each beam is discretized into 10 Finite Elements. Structural damage is simulated by reducing the height of selected finite elements, as illustrated in Fig. 1. DynamicFEA is performed for four damage scenarios, involving damage to one, two, three or fourFinite Elements, respectively. For each case, all possible combinations of damage locations are considered, corresponding to the binomial combinations $^{n}C_{r}$where n=10 is the total number of finite elements and r=4 is maximum number of damaged finite elements location. Total damages considered are $\sum_{r}\ ^{n}C_{r}$ where r = 1, 2, 3, 4 and n = 10.

The currentresearch demonstrates that the elastic addition of a small auxiliary mass (as illustrated in Fig. 1) enhances the predictive accuracy of the ANN. For each increment of the added mass, modal amplitude values were numerically computed via free vibration Dynamic FEA. From the resulting dataset, only 10 (not all) modal amplitude values per beam were selected as inputs to the ANN for each of the four beam configurations. Prior to being fed into the ANN, these modal amplitude values underwent pre-processing. The proposed quadruple–loss ANN architecture successfully predicts both the location and magnitude of multiple damage scenarios, with damage sizes ranging from 0.1 mm to 0.3 mm.

3. Methodology

The core concepts underpinning the current research are as follows:

1. Introduce a small auxiliary mass elastically to finite elements (FE) of 1-D beam in order to enhance the sensitivity of modal amplitude data.
2. Generate extensive modal amplitude datasets by incrementally striding a small-mass strip along the length of the 1-D beam.
3. Simultaneously minimize Wasserstein Distance (WD), Jensen-Shannon Divergence (JSD), Mean Squared Error (MSE), and Mean Absolute Error (MAE) loss functions using a Predictor–Critic ANN regression framework.
4. Transform the estimated damage levels of FE into a SoftMax-based probability mass function (PMF), which is subsequently employed to minimize WD, JSD, and MSE.
5. Employ a SVM classifier for damage identification.
6. Utilize a Predictor Neural Network to estimate both the location and amount of structural damage.

3.1 Divergence for Probability Mass Function (PMF)

3.1.1 KullbackLeibler Divergence (KLD) – The KLD quantifies the dissimilarity between two probability distributions. When employed as a loss function in ANNs, it measures the informational cost required to align the predicted distribution (Q) with the target distribution (P). If the random variable is X

$$KLD(P \mid\mid Q) = \sum_{x \in X} P(x) \times \log[P(x)/Q(x)]$$

KLD is not symmetric.

$$KLD(P \mid\mid Q) \neq KLD(Q \mid\mid P)$$

If two probability distributions are far apart horizontally, then KLD suffers a divide by zero (or divide by a very small number) problem. If distributions P and Q are not overlapping, then for given x where $P(x) \neq 0$then $Q(x) = 0$, in that case, $KLD(P \,||\, Q) = \infty$. If distribution P and Q are slightly overlapping, then for given x,the $Q(x) \sim 0$, then $KLD(P \,||\, Q) \sim \infty$, it means KLD will be a large number and it will cause explosion of gradient during Backpropagation. If $LD(P \,||\, Q) \sim 0$, it means ANN approximated predicted PMF (Q) to target PMF (P).

3.1.2 Jensen Shannon Divergence (JSD) – JSD is a modification over KLD. Like KLD, the JSD describes how dissimilar two probability distributions are.

$$JSD(P \,||Q) = \frac{1}{2}KLD(P \,||\, M) + \frac{1}{2}KLD(Q \,||\, M)$$

Where Mixture Distribution $M = \frac{1}{2}\,(P + Q)$

JSD is symmetric, $JSD(P \,||Q) = \; JSD(Q \,||\, P)$

When JSD is used as a loss function in ANN, it describes the effort required to make predicted PMF (Q) similar to target PMF (P). JSD, when used as a loss function in ANN, avoids the divide by zero (or divide by very small number) problem; it helps prevent gradient explosion during Backpropagation. This is the reason to use JSD as one of the loss functions for Predictor Neural Network Regression in current research. If $JSD(P \,||\, Q) \sim 0$, it means ANN approximated predicted PMF (Q) to target PMF (P).

3.1.3 Wasserstein Distance (WD) – The Wasserstein distance, also known as the *Earth Mover's Distance* (EMD) in computational contexts, provides a metric to quantify the dissimilarity between two probability distributions over a given metric space.

Formally, let $(\mathbb{X}, d)$ be the Polish metric space, and let $\mu, v \in P_p(\mathbb{X})$ be two probability measures with P^{th} finite moment, i.e.

$\int_{\mathbb{X}} (x_0, x)^p d\mu(x) < \infty, \int_{\mathbb{X}} (x_0, y)^p dv(y) < \infty$

For some $x_0 \in \mathbb{X}$and $p \geq 1$. The P–Wasserstein Distance is defined as

$$W_p(\mu, v) := \left(inf_{\gamma \in \Gamma(\mu, v)} \int_{\mathbb{X} \times \mathbb{X}} d(x, y)^p d\gamma(x, y) \right)^{1/p}$$

Where $\Gamma(\mu, v)$ denotes the sets of all coupling γ on $\mathbb{X} \times \mathbb{X}$ with marginals $\mu$and v.

The Wasserstein Distance thus represents the minimum cost of transporting mass from $\mu$to v, where the cost of moving unit mass from x to y is given by $d(x, y)^p$.

The special case where $\mathbb{X} = \mathbb{R}^d$ and $d(x, y) = \| x - y \|$ is the Euclidian Distance, the Wasserstein metric induces a meaningful topology $P_p(\mathbb{R}^d)$, strictly stronger than weak convergence but weaker than total variation.

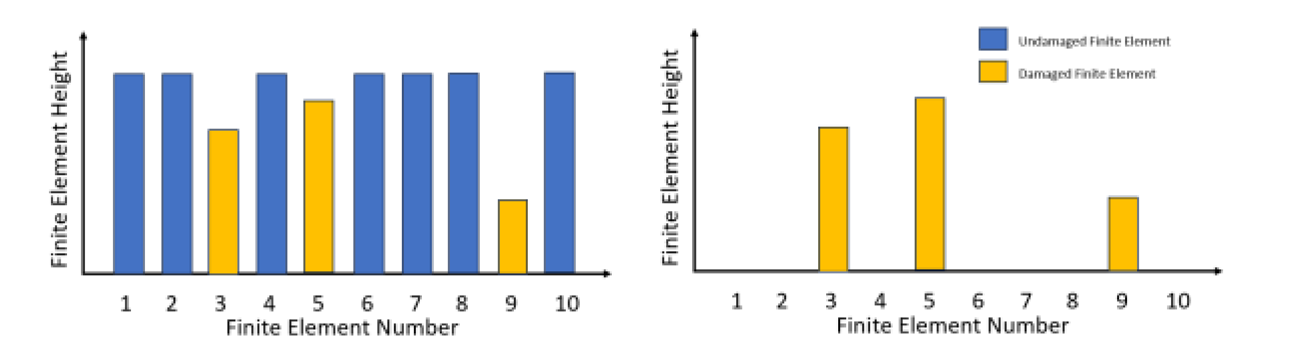
3.2 Damage Amount as PMF

SoftMax (S) is a very common activation function for ANN classification problems, and it is used with Categorical Cross-Entropy loss function usually. If HD is Height of Damages Finite Element,

$$S(HD_i) = \frac{e^{HD_i}}{\sum_j e^{HD_j}}$$

$\sum_j S(HD_j) = 1$and $0 \leq S(HD_i) \leq 1$, is sufficient conditions to consider S as a PMF.

$HD_i$is the height of ith damaged finite element, and j represents the total number of damaged finite elements and e is Euler's Number.



(a) (b)

Fig. 3. Damaged FE height to SoftMax probability (a) All FE heights (b) Only damaged FE heights

Current paper minimizes WD and JSD between two PMFs, a target PMF and a PMF predicted by Predictor ANN. For target PMF height of only damaged finite elements (HD) are considered (Fig. 3.b) and then converted into SoftMax Distribution $S(HD_i)$ as described above.

When calculating target PMF, we consider the undamaged FE height as zero and only consider the heights of damaged FE'sHD_i (Fig. 3.b), because the objective is to find out the location of damage and the height of the damaged FE. This approach makes the objective simpler and easy to converge. Since the output layer of Predictor ANN (Fig. 4) uses SoftMax activation, the predicted PMF is also SoftMax distribution. Predicted SoftMax PMF is approximated with target SoftMax PMF by reducing WD, JSD and MSE simultaneously by Predictor ANN.

3.3 Data Preprocessing for ANN Regression

1. The free vibration Dynamic FEA is formulated as an eigenvalue problem. The resulting eigenvectors represent the modal amplitude values, and these vectors are normalized to rescale.
2. During the data preprocessing phase, the normalized modal amplitude values are further standardized to have zero mean and unit standard deviation. These standardized vectors serve as input features to the Predictor and Critic ANN.
3. Modelling the structural damage, the heights of undamaged FE are assigned a value of zero, while the original heights of damaged elements are retained (Fig. 3). The resulting height vector, whose size corresponds to the number of damaged elements, is transformed into a SoftMax-based probability PMF. This PMF serves as one of the target outputs for the Predictor ANN, which is trained by minimizingWD, JSD and MSE loss functions, as illustrated in Fig. 4.
4. The physical units used for the 1-D beam model are: mass in grams, length in millimetres, and time in seconds.
5. Structural damage is introduced by reducing the height of selected FE within the range of 0.1 mm to 0.3 mm.

Damage identification in1D beams is formulated as a binary classification problem, distinguishing between 'damage' and 'no damage' classes. A SVM with a Gaussian kernel is trained using pre-processed modal amplitude data labelled according to these two classes. To address class imbalance, where 'no damage' instances dominate, artificial noise sampled from a uniform distribution is added to the 'no damage' class. This approach aligns with standard practices for handling imbalanced datasets by introducing noise, as proposed byDackermann et al. [9]. Importantly, uniform rather than Gaussian noise is used to prevent any correlation, because the SVM uses of a Gaussian kernel. The trained SVM achieves a classification accuracy of 99.99% across all four boundaries of 1-D beams as shown in Fig. 2.

3.4 ANN Architecture

In the damage identification framework for a 1-D beam using FE modelling, the heights of the damaged FE are treated as one of the primary targets for a deep ANN. To represent these heights in a probabilistic manner, they are transformed into a PMF using the SoftMax function. This SoftMax-based distribution effectively highlights the relative likelihoods of damage across the FE, and is used as one of the output targets of the ANN. In addition to this probabilistic representation, the Predictor ANN is also trained to predict the actual heights of the damaged FE directly, thus making it a multi-target learning problem.

There are two ANNs, one is Predictor Network and another is Critic Network.WD is one of the losses minimized. Optimizing WD (3.1.3) is intractable problem to solve. Kantorovich–Rubinstein duality (Ref.[20],) is a fundamental result in optimal transport theory that provides an alternative formulation of the first order WD.

Further explaining 3.1.3

$$W_1(\mu, v) := sup_{||f||_L \leq 1}\left(\int_{\mathbb{X}} f(x)d\mu(x) - \int_{\mathbb{X}} f(x)dv(x)\right)$$

$$W_1(\mu, v) := sup_{||f||_L \leq 1}\left(\mathbb{E}_{x\sim\mu}[f(x)] - \mathbb{E}_{x\sim v}[f(x)]\right)$$

$||f||_L \leq 1$represents 1-Lipschitz continuity.

Martin Arjovsky et al. [21] proposed dual ANN model to implement 1-Lipschitz continuity by clipping weights of ANN in very small region. This method suffers from Capacity underuse, Vanishing gradients, Poor generalization because of weight clipping in very small region to satisfy 1-Lipschitz continuity.

Alternate to weight clipping was proposed by Ishaan Gulrajani et al. [22] with dual ANN model. Instead of clipping weights in very small region, proposal is to penalize the norm of gradient of the critic with respect to its input. Predictor-Critic dual ANN model in this paper is inspired from this approach. Gradient penalty approach proposed by Gulrajani et al. [22] has been used by Zhi-Dong Li et al. [23] for transfer learning-based damage identification with FEA for real structures.

Combined Predictor-Critic dual ANN model minimizes first order WD between two PMFs, target PMF is damaged heights of FE which are converted into SoftMax probabilities and corresponding predicted PMF (there is SoftMax activation in last layer of ANN). Predictor ANN also minimizes JSD. Minimizing first order WD and JSD together basically reduces divergence of two PMFs horizontally and vertically. Third loss function in Predictor ANN is MSEbetween target PMF and predicted PMF, this gives stability to learning process. Fourth loss function in Predictor ANN is MAE between actual heights of damaged FE and predicted heights of damaged FE. These four losses are minimized simultaneously in this way they work in tandem. For example, if MAE reaches near zero then MSE, JSD and WD will also be near to zero and vice-versa. If one of the four losses goes down then it helps other three loss functions to minimize. In this way the four loss functions work in tandem.
Algorithm – 1 describes training procedure of Predictor – Critic dual ANN.

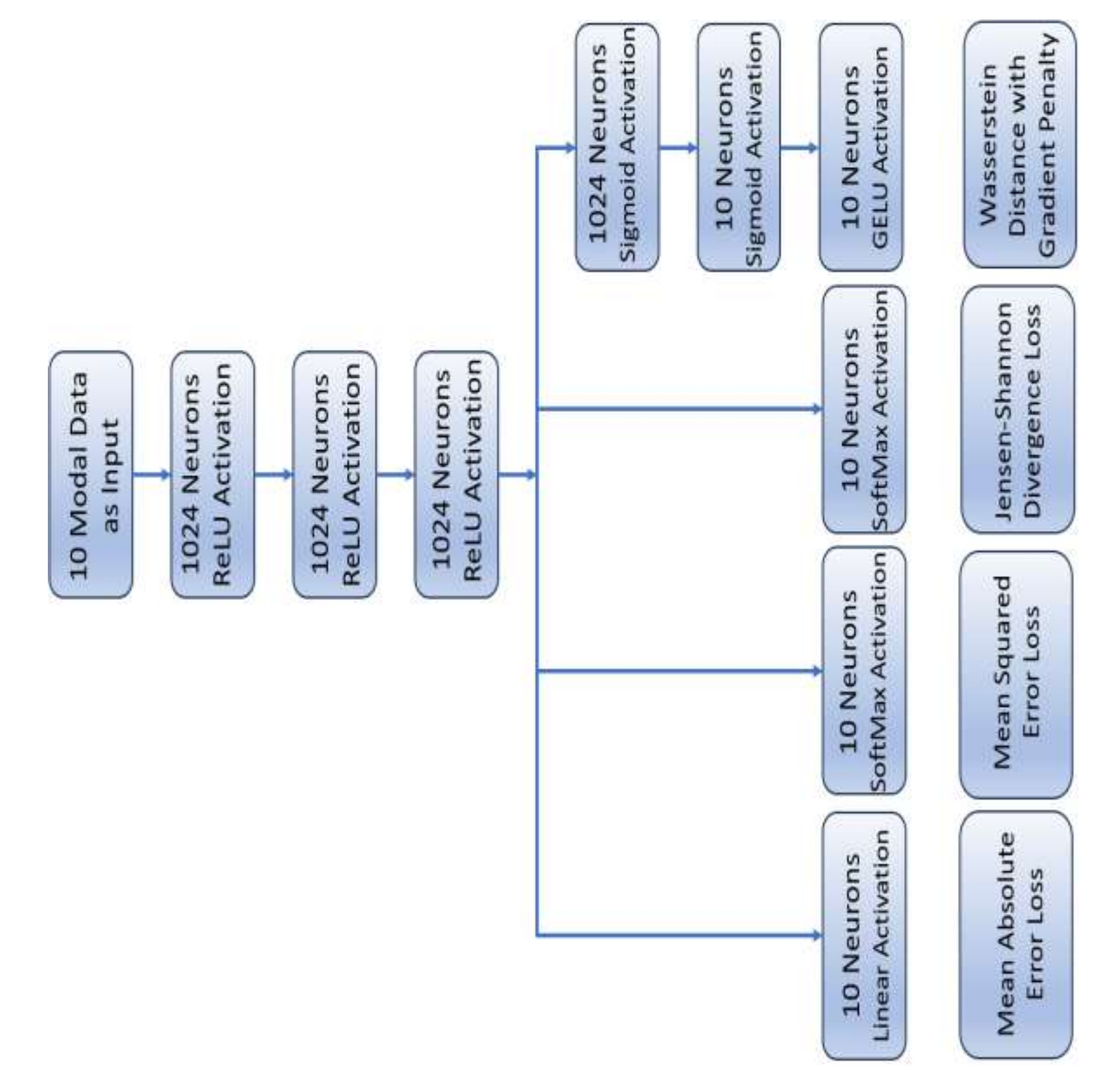


Fig. 4. Predictor Artificial Neural Network

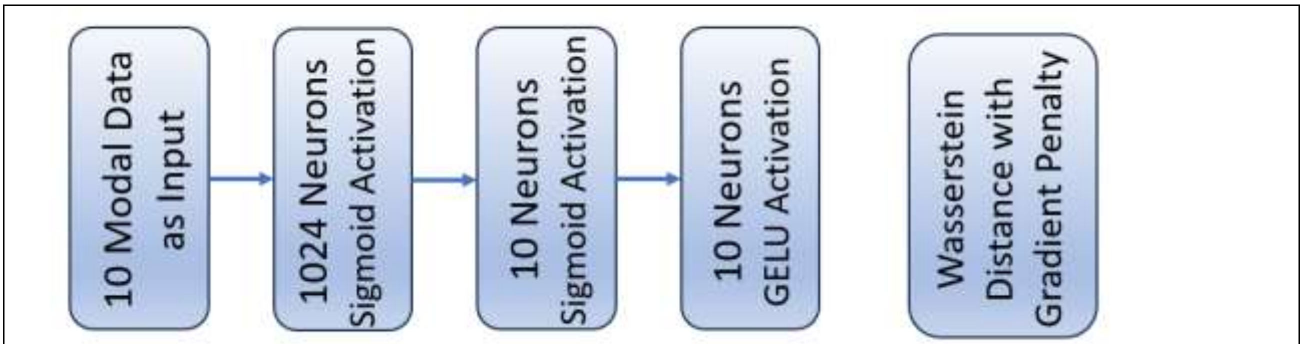


Fig. 5. Critic Artificial Neural Network

Algorithm – 1 Training Procedure for Predictor – Critic dual ANN
Required: Modal Amplitude data m, SoftMax Probabilities of Damaged Finite Element Heights s, Actual Height of Damaged Finite Element d, Critic Steps N_{critic}, Predictor Network p, Learnable Parameters Ω of p, Critic Network C, Learnable Parameter ø of c, learning rate η_p of p, learning rate η_c of c, Gradient Penalty g, Wasserstein Distance w with g, Ground Truth of Jensen-Shannon Divergence Loss is j, Ground Truth of Mean Squared Error Loss is l, Ground Truth of Mean Absolute Error Loss is A, Total Loss for p is t, Number of Epochs N_{epochs}, Hyper Parameter € for Interpolation, Gradient Penalty Coefficient λ, JSD is Jensen-Shannon Loss Function, MSE is Mean Squared Loss Function, MAE is Mean Absolute Error Loss Function, The Values $\tilde{x}$, $\hat{x}$and x corresponds to different values of Wasserstein Distance, Underscore '_' is used to ignore one of the outputs from multiple outputs of Predictor Network p.
for 1,⋯,N_{epochs} for m, s, d in Dataset for 1,⋯, N_{critic} $\tilde{x}$,_,_,_ = p(m) $\hat{x}$= C(m) $x \leftarrow €\hat{x} + (1 - €)\tilde{x}$ G= $\lambda(\Vert \nabla_x C(x) \Vert_2 - 1)^2$ w =$\tilde{x} - \hat{x}$ +G ø← $Adam$ (w, ø, η_c) end for _, J, L, A = p(m) j = JSD(J, s) l = MSE(L, s) A = MAE(A, d) $\hat{x}$= C(m) t= $\hat{x}$ +j+l+A Ω← $Adam$ (t, Ω, η_p) end for end for

3.5 ANN Hyperparameters

Following are the hyperparameter settings

1. JSD loss and MSE loss are scaled up by a factor of 10, and MAE loss is scaled down by a factor of 0.1, Gradient Penalty parameter is 0.3 to calculate WD with Gradient Penalty.
2. Total loss for Predictor Network is the sum of JSD, MAE, and MSE losses along with WD with Gradient Penalty.
3. 200 Epochs are taken for training Predictor-Critic dual ANN, batch size is taken as 32, Critic Step N_{critic} is 3

Adam Optimizer, proposed by Kingma [19], is used withlearning rate for Predictor Network as 0.001 and learning rateis 0.0005 for Critic Network. Other parameters of Adam optimizer are set to proposed defaults by Kingma [19].

ANN is trained on sensitive Modal Amplitude data produced by a small striding strip of material, but it is validated and tested on Modal Amplitude data produced without mass increment. It makes the proposed methodology practical because it is impractical to stride strip over the engineering structure in real life. Best ANN learnable parameters are saved for minimum validation JSD loss on validation data, and the proposed methodology uses MAE loss on test data as a metric to evaluate the ANN regression. This decoupling between saving the ANN learnable parameters and evaluating the performance of the ANN regression model avoids any room for bias that may exist. The MAE loss on test data also works as a metric to find the best value of mass increment that returns the minimum value of MAE.

The learning (total loss) curves (Fig. 6. to Fig. 9.), considered for different beams, indicate no overfitting during the learning process and justifythe ANN architecture, hyperparameters and data preprocessing.

Fig. 6.toFig. 9. indicate that total loss during training and total loss on validation data goes down very well for both no mass increment and with mass increment case.

4. Results and Discussions

The results of MAE lossfor 1-D beam with 4 different boundary conditions are shown in Table 1. MAE is one of the four loss functions considered in Predictor Neural Network as shown in Fig. 4. The objective is to find damage amount.The MAE between heights of actual damaged FE and predicted damaged FE is minimized by Predictor Neural Network. This is the reason MAE is right metric to decide optimal percentage mass increment in FE, which is hyperparameter to finetune. Additional mass is added to FE elastically by incrementing its density between 0.08% to 50%. Optimal value of mass increment is decided by minimum value of MAE.

Table 1: MAE for different mass increments

MAE of Damaged FE Heights					
Beam Type		**Cantilever**	**Simply Supported**	**Overhanging**	**Propped Cantilever**
Percentage Increment in FE Mass	**0**	0.01196035	0.00171830	0.00433335	0.00539908
	0.08	0.00122971	0.00008758	0.00015379	0.00064985
	0.09	0.00120485	0.00008619	0.00023553	0.00078849
	0.1	0.00106195	0.00007561	0.00016291	0.00049406
	0.5	0.00178445	0.00019099	0.00012153	0.00113565
	1	0.00176054	0.00008130	0.00023647	0.00113939
	2	0.00338424	0.00012877	0.00016549	0.00263384
	3	0.00584349	0.00015693	0.00019444	0.00213273
	4	0.01110954	0.00012681	0.00029786	0.00300547
	5	0.01679285	0.00013924	0.00053874	0.00317727
	10	0.01902758	0.00017721	0.00148453	0.00509250
	20	0.01221385	0.00038028	0.00439957	0.02843057
	50	0.02780659	0.00267103	0.00541639	0.25510502

It can be seen in Table 1 that when there is no mass increment i.e. 0% mass increment, the MAE is high. As the mass is increased by amount of 0.08% the MAE starts reducing and it is true for all four boundary conditions. As the mass increment reaches 50% the MAE is higher than the MAE of 0% mass increment. Optimal value of percentage mass increment is in between 0% mass increment and 50% mass increment.

Optimal values (from Table 1) for all the four boundary conditions are as followed.

1. For Cantilever Beam, is it 0.1% mass increment
2. For Simply Supported Beam, it is 0.1% mass increment
3. For Overhanging Beam, it is 0.5% mass increment
4. For Propped Cantilever Beam, it is 0.1% mass increment

The Predictor – Critic ANN are trained (Algorithm – 1) for these optimal mass increment values. The learning curves (total loss) for Predictor ANN which is used to localize and quantify the damage in beam are shown in Fig. 6.toFig. 9. Total loss is sum of Wasserstein Distance Loss, Jensen Shannon Divergence Loss, Mean Squared Error Loss and Mean Absolute Error Loss. The Predictor ANN is trained for 200 epochs and the total loss for training and validation data is presented in Fig. 6.toFig. 9.

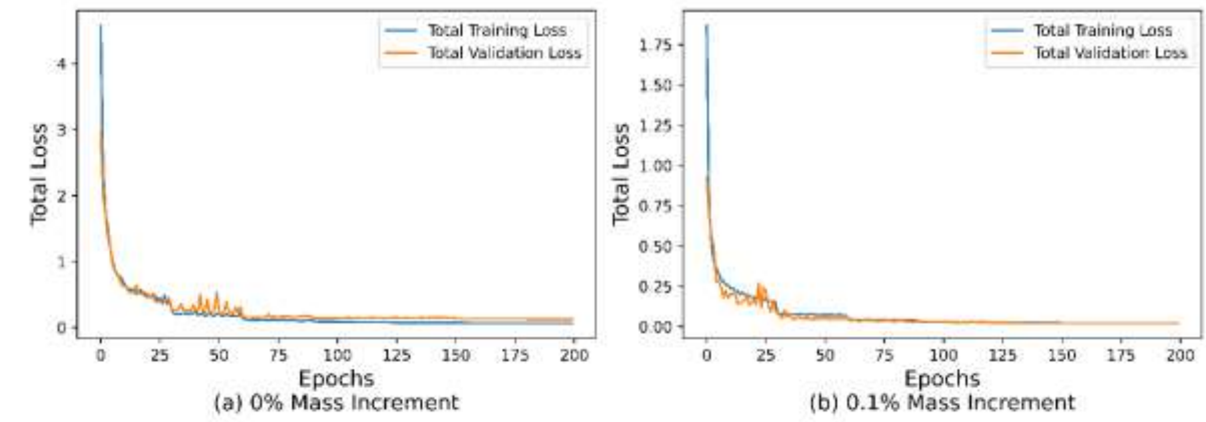


Fig. 6. Cantilever Beam Learning curves (a) 0% mass increment (b) 0.1% mass increment

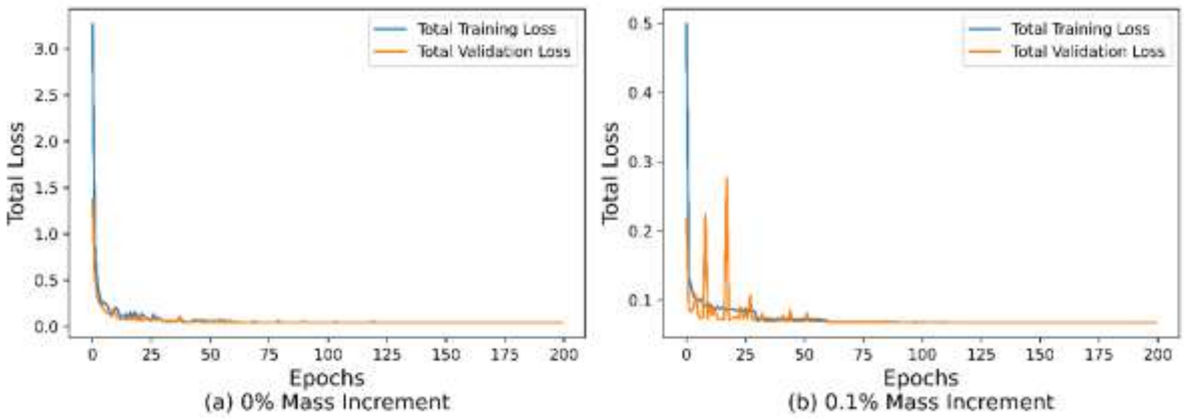


Fig. 7. Simply Supported Beam Learning curves (a) 0% mass increment (b) 0.1% mass increment

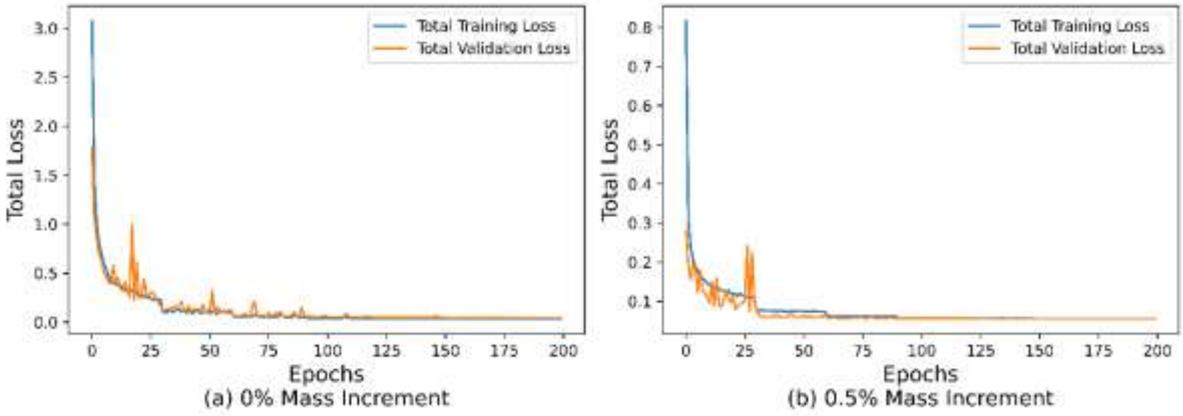


Fig. 8. Overhanging Beam Learning curves (a) 0% mass increment (b) 0.5% mass increment

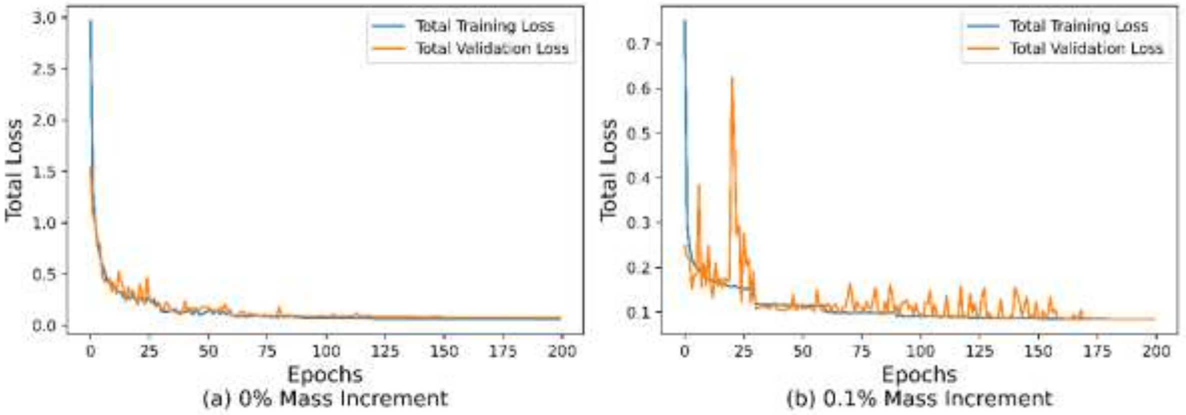


Fig. 9. Propped Cantilever Beam Learning curves (a) 0% mass increment (b) 0.1% mass increment

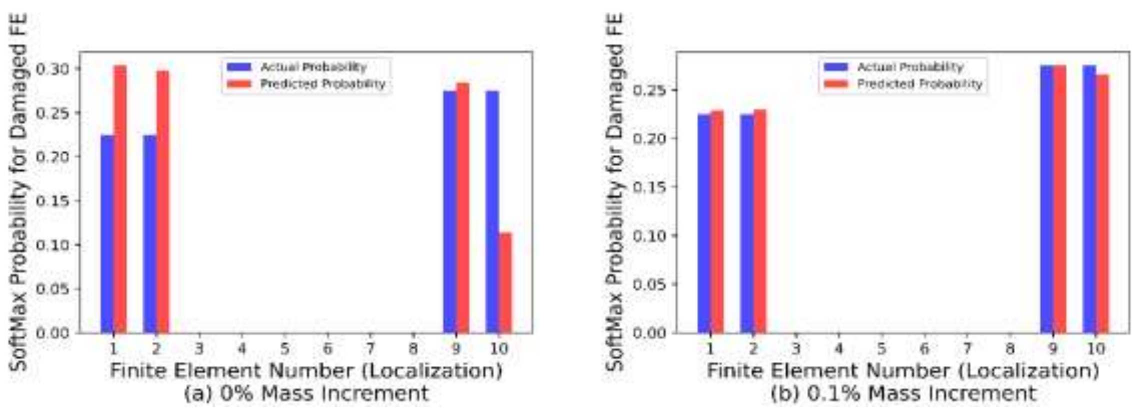
It is clear from the learning curves (Fig. 6. to Fig. 9.) that total training and validation losses are reduced to near zero values. After 200 epoch there is no offset in total training loss and total validation loss which is evidence that there is neither Overfitting nor Underfitting.

There are four output layers in Predictor ANN (Fig. 4). Second output reduces the SoftMax probabilities by reducing JSD. Since there are 10 FE considered it means there are 10 possible locations of damages in beam. Damage locations are varying from 1 damage to 4 damages and identifying 4 damages simultaneously is relatively most difficult. Location of spike of SoftMax probabilities are indication of locations of damages present in beam.

The fourth output of Predictor ANN (Fig. 4) reduces the MAE between actual and predicted heights of damaged FE. Since there are 10 FE considered it means there are 10 possible locations of damages in the beam. Damage locations are varying from 1 damage to 4 damages and identifying 4 damages simultaneously is relatively most difficult. Location of spike of FE heights are direct measurement of amount of damage present in beam. Ideally the locations of spike in SoftMax probabilities and spike of FE heights should remain same, hence it is double check of location of damage in the beam.

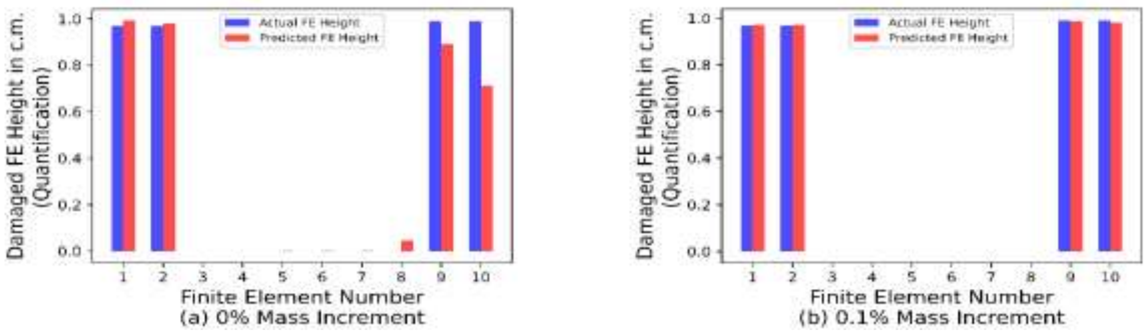
Along with Fig.10 to Fig. 17 the values of JSD and damaged FE heights along with MAE are compared for no mass increment and with mass increment case for all four boundary conditions of 1-D beams.

Fig. 10 to Fig. 17 shows the SoftMax probability and damaged FE height for all four boundary conditions. It can be seen that, Fig. 10 to Fig. 17 infer that Predictor ANN accurately predicts the location and quantity of the damage better when there is a small amount of mass addition.



Cantilever Beam

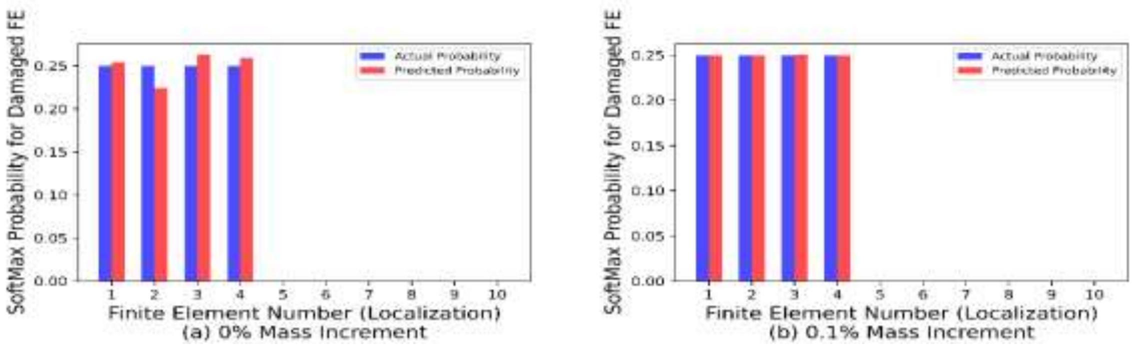
Fig. 10. SoftMax probabilities(a) 0% mass increment (b) 0.1% mass increment



Cantilever Beam

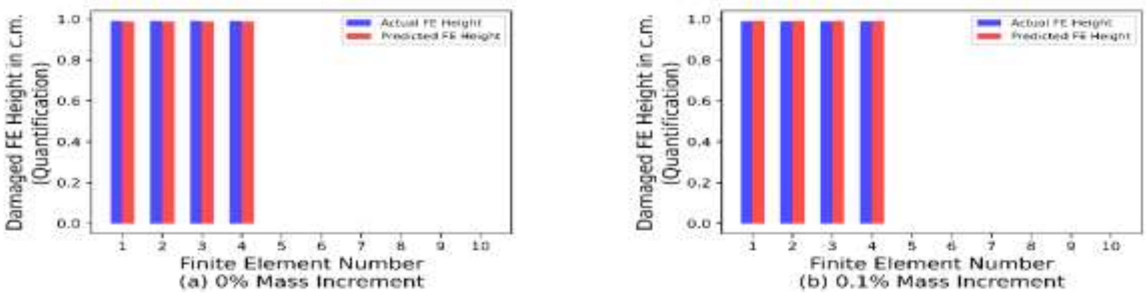
Fig. 11. Damaged FE heights (a) 0% mass increment (b) 0.1% mass increment

For Cantilever Beam (Fig. 10 and Fig. 11) FE 1, 2, 9 and 10 are damaged and actual FE heights are 0.97, 0.97, 0.99, and 0.99 cm. In the case of 0% mass increment, there is a JSD of 0.032731, and in the case of 0.1% mass increment, there is a lower value of JSD, which is 0.000083, it is first numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data. Predicted FE heights in case of 0% mass increment are 0.99264, 0.98113, 0.89244 and 0.71280 cm. In the case of 0.1% mass increment, predicted heights are0.97195, 0.97214, 0.98854 and 0.98261 cm, which is better. It is second numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data. In the case of 0% mass increment, the MAE is 0.045927, but the 0.1% mass increment case has a much lower MAE of 0.001321. It is third numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data. In Fig. 11 (a) the wrong spike of FE height at location 5, 6, 7 and 8 can be seen for 0% mass increment, which is not present in Fig. 11 (b) for 0.1% mass increment.



Simply Supported Beam

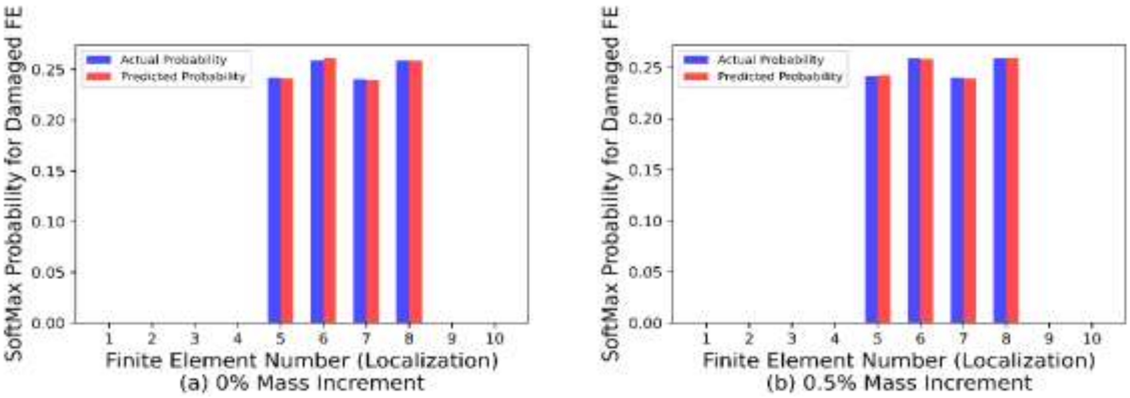
Fig. 12. SoftMax probabilities(a) 0% mass increment (b) 0.1% mass increment



Simply Supported Beam

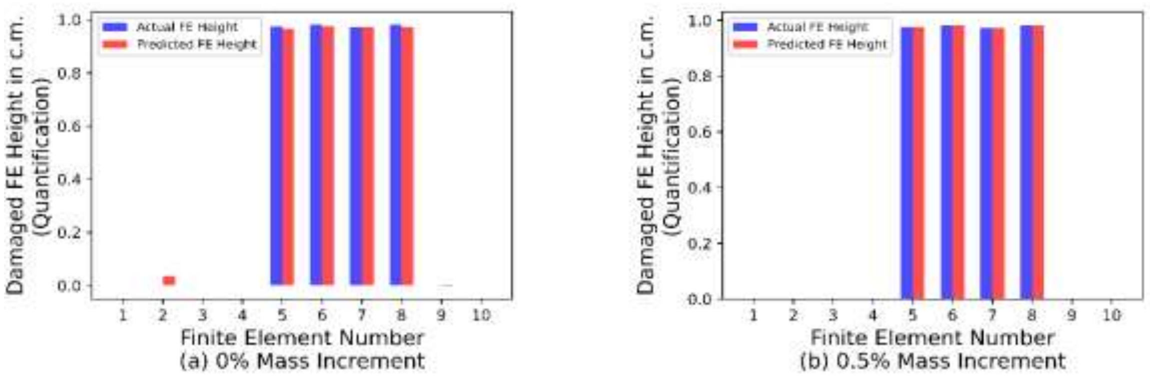
Fig. 13. Damaged FE heights (a) 0% mass increment (b) 0.1% mass increment

For Simply Supported Beam (Fig. 12 and Fig. 13) FE 1, 2, 3 and 4 are damaged and actual FE heights are 0.99, 0.99, 0.99, and 0.99 cm. In the case of 0% mass increment, there is a JSD of 0.000714, and in the case of 0.1% mass increment, there is a lower value of JSD, which is 0.000001, it is first numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data. Predicted FE heights in case of 0% mass increment are 0.98666, 0.98879, 0.98648 and 0.98908cm. In the case of 0.1% mass increment, predicted heights 0.99027, 0.99002, 0.99032 and 0.99006cm, which is better. It is second numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data. In the case of 0% mass increment, the MAE is 0.001099, but the 0.1% mass increment case has a much lower MAE of 0.000079. It is third numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data



Overhanging Beam

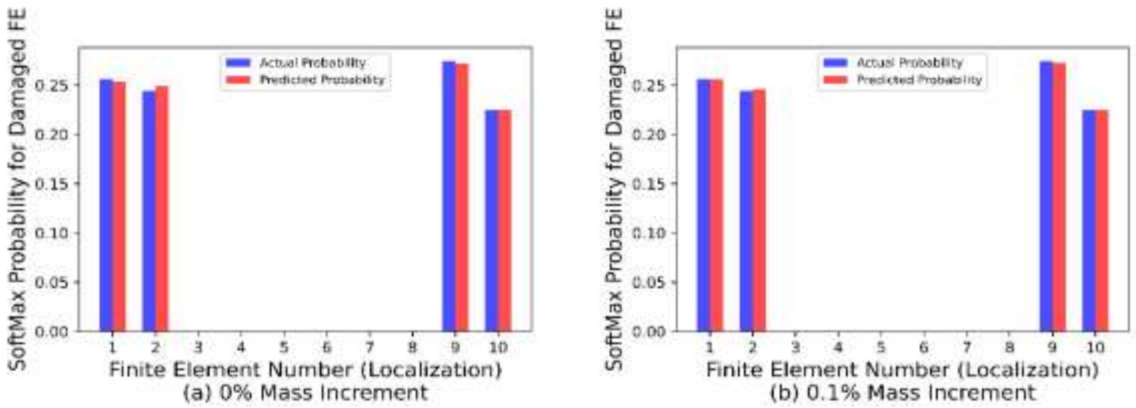
Fig. 14. SoftMax probabilities(a) 0% mass increment (b) 0.5% mass increment



Overhanging Beam

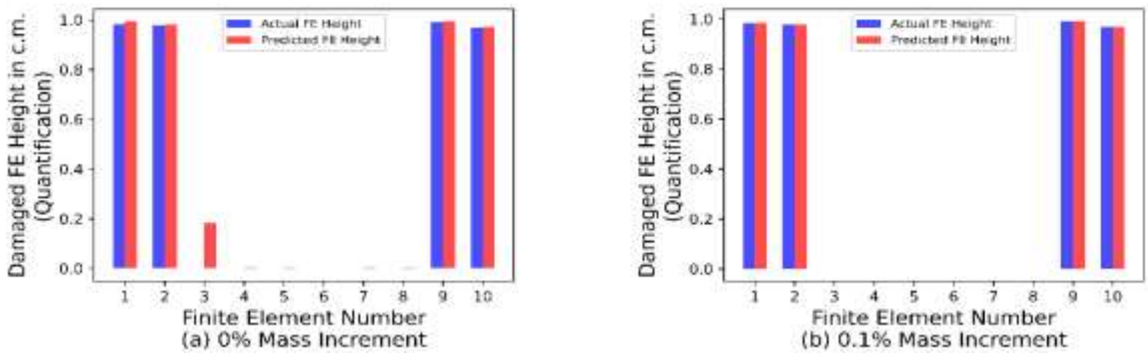
Fig. 15. Damaged FE heights (a) 0% mass increment (b) 0.5% mass increment

For Overhanging Beam (Fig. 14 and Fig. 15) FE 5, 6, 7 and 8 are damaged and actual FE heights are 0.97430, 0.98120, 0.97360 and 0.98120 cm. In the case of 0% mass increment, there is a JSD of 0.000028, and in the case of 0.5% mass increment, there is a lower value of JSD, which is 0.000001, it is first numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data. Predicted FE heights in case of 0% mass increment are 0.96687, 0.97583, 0.97379 and 0.97371cm. In the case of 0.5% mass increment, predicted heights 0.97457, 0.98086, 0.97331 and 0.98080 cm, which is better. It is second numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data. In the case of 0% mass increment, the MAE is 0.006221, but the 0.5% mass increment case has a much lower MAE of 0.000134. It is third numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data. In Fig. 15 (a) the wrong spike of FE height at location 2 and 9 can be seen for 0% mass increment, which is not present in Fig. 15 (b) for 0.5% mass increment.



Propped Cantilever Beam

Fig. 16. SoftMax probabilities(a) 0% mass increment (b) 0.1% mass increment



Propped Cantilever Beam

Fig. 17. Damaged FE heights (a) 0% mass increment (b) 0.1% mass increment

For Propped Cantilever Beam (Fig. 16 and Fig. 17) FE 1, 2, 9 and 10 are damaged and actual FE heights are 0.98320, 0.97840, 0.99 and 0.97 cm. In the case of 0% mass increment, there is a JSD of 0.00003, and in the case of 0.1% mass increment, there is a lower value of JSD, which is 0.000005, it is first numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data. Predicted FE heights in case of 0% mass increment are 0.99328, 0.98030, 0.99308 and 0.97294 cm. In the case of 0.1% mass increment, predicted heights 0.98352, 0.97914, 0.99010 and 0.97050 cm, which is better. It is second numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data. In the case of 0% mass increment, the MAE is 0.020245, but the 0.1% mass increment case has a much lower MAE of 0.000188. It is third numerical evidence to support that increasing FE mass slightly improves sensitivity of modal amplitude data. In Fig. 17 (a) the wrong spike of FE height at location 3, 4, 5, 7 and 8 can be seen for 0% mass increment, which is not present in Fig. 17 (b) for 0.1% mass increment.

It is established from three numerical evidences namely JSD, MAE and predicted damaged FE heights that slight increase in mass improves the prediction of damage location and damage amount as compared to no mass addition. It means adding small amount of mass elastically improves the sensitivity of modal amplitude data which leads to better predictions.

5. Conclusions

The proposed Predictor–Critic ANN architecture (Fig. 4 and Fig. 5), employing consistent hyperparameters and uniform data preprocessing across all four beams, effectively mitigates overfitting during training. During the testing phase, the ANN accurately achieves both damage localization and damage quantification. The methodology is validated using four beams with identical geometrical dimensions and material properties but differing boundary conditions, thereby demonstrating robustness of Algorithm.

The present study provides three numerical indicators demonstrating that the sensitivity of modal amplitude data improves with a slight increase in mass. For all four beams investigated:

1. Predicted damaged FE heights are better for cases with a small mass increment compared to the nomassincrement case.
2. The JSD values are decreased when a small mass increment is introduced, relative to the case without mass increment.
3. The MAE between the actual damaged FE heights and the corresponding predicted FE heights are reduced in the presence of a small mass increment compared to the nomassincrement case.

The current research can be used as CBMS (Appendix B)

Future Scope: The present work can be extended to beam, plate, and shell structures containing internal cracks, facilitating accurate identification of concealed damage. The inclusion of damping effects is expected to further enhance the accuracy of finite element models. Laboratory-scale experimental studies on representative engineering structures can be conducted to calibrate the finite element model and determine the optimal mass increment. The resulting refined methodology may then be applied to full-scale engineering structures, enabling reliable detection and quantification of internal cracks, which is essential for ensuring structural safety and effective maintenance

Conflict of Interest: There is no conflict of interest among authors.

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Appendix A:

Each of the four 1-D beams considered have total 20 Degrees of Freedom (DOF) that's why there are 20 possible modal amplitude values. The current work considers only 10 Modal Amplitude values corresponding to first natural frequency that's why the data considered is 10-Dimensional.

The current research considers all binomial combinations $^{n}C_{r}$, where n=10 is the total number of finite elements and r=4 is number of damaged FE locations. Total damages considered are $\sum_{r} {}^{n}C_{r}$ where r = 1, 2, 3, 4 and n = 10. Here in this section n = 10 and r = 1, means only one damage per FE is considered, it means data has 10 classes to classify (10 damaged FE locations). Two scenarios i.e. no mass addition and addition of mass elastically are considered for all four boundary conditions of 1-D beam.

If modal amplitude data, which has 10 classes, will be classified with great accuracy, if it has 10 cohesive clusters. 10-Dimensional cluster can be visualized if projected in 2-Dimensional space, conditioned; if local relationship and local structure of 10-Dimensional space are preserved in 2-Dimensional space. The t-distributed Stochastic Neighbor Embedding (t-SNE) algorithm, Maatenet al. [24], is considered here for visualization because it preserves the local relationship and local structure of higher dimensional space in lower dimensional space. Fig. 1A. to Fig. 4A. is 2-Dimensional representation of 10-Dimensional Modal Amplitude data by t-SNE algorithm.

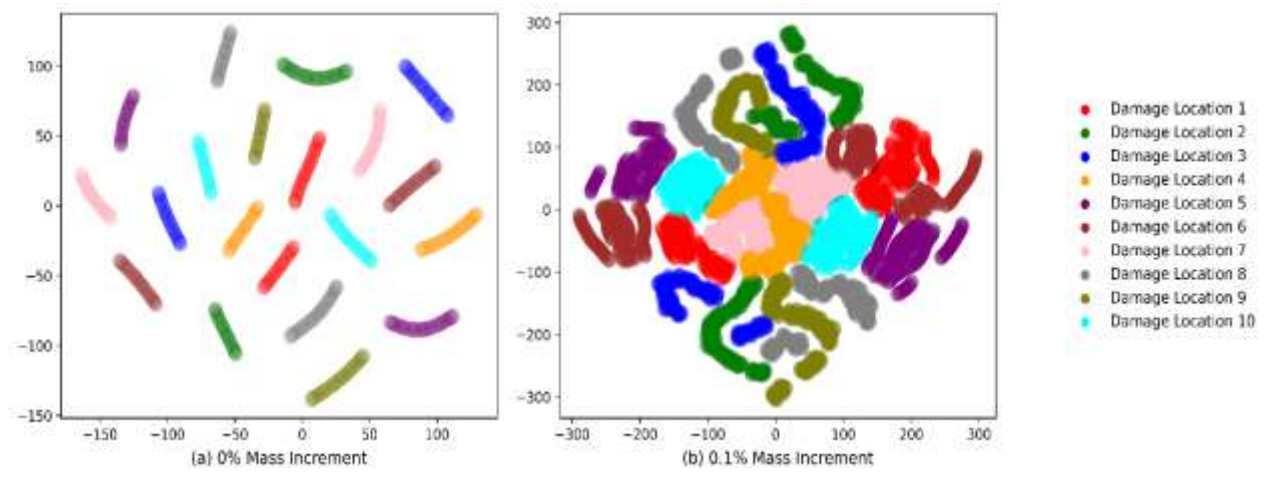

Fig. 1A. 2-D representation of Cantilever Beam Modal Amplitude Data

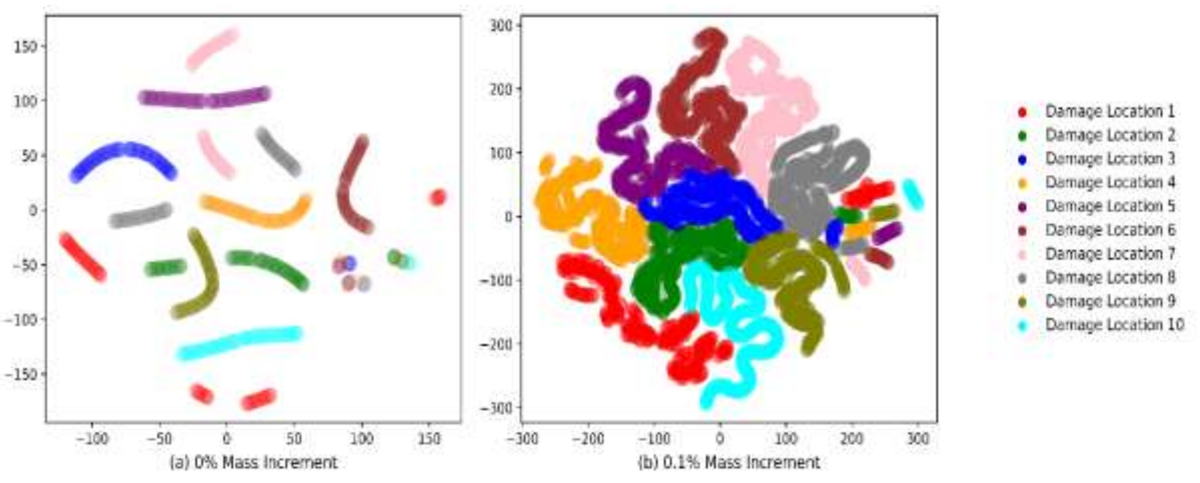

Fig. 2A. 2-D representation of Simply SupportedBeam Modal Amplitude Data

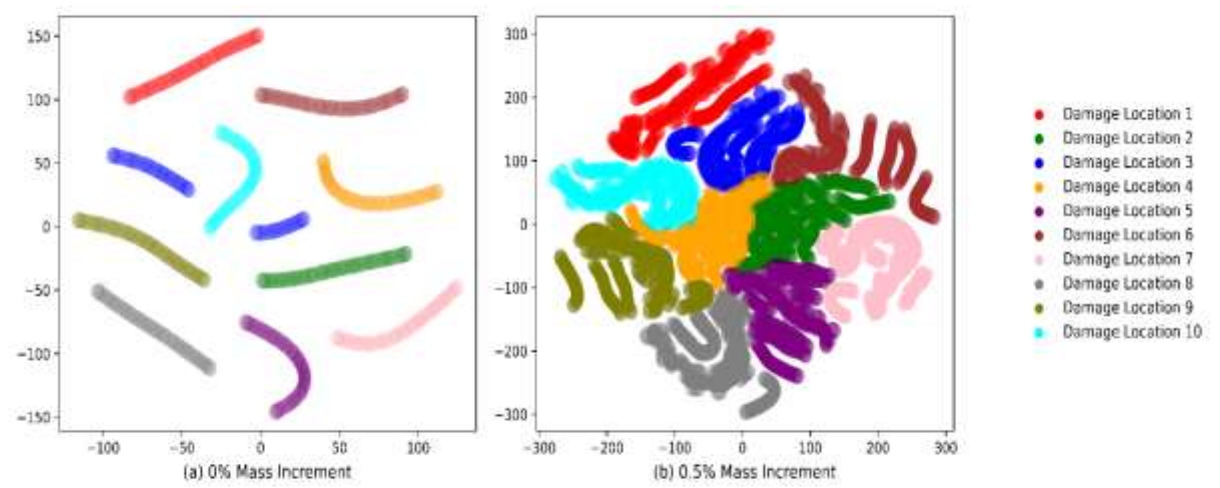

Fig. 3A. 2-D representation of OverhangingBeam Modal Amplitude Data

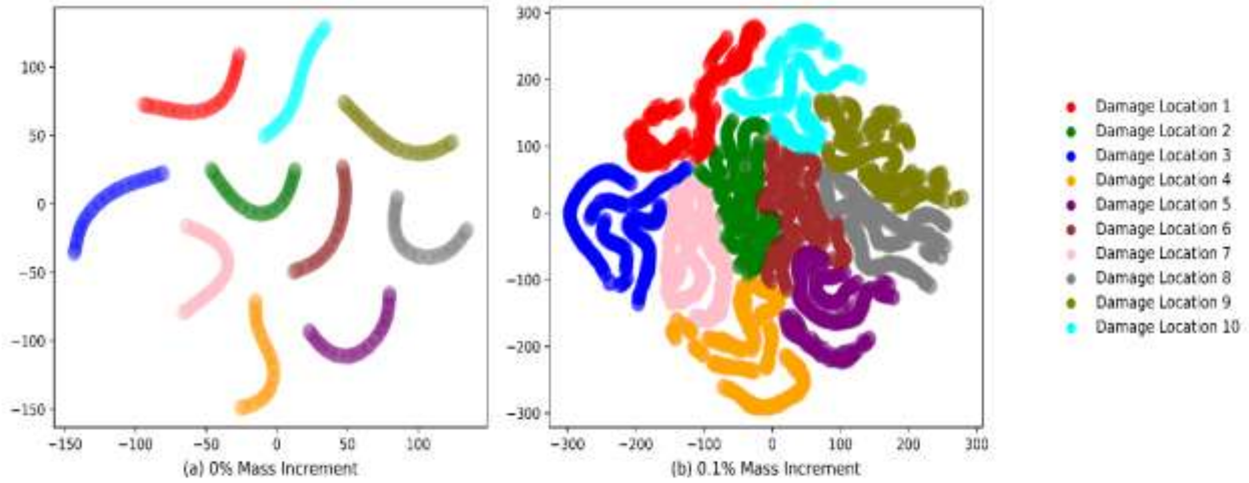

Fig. 4A. 2-D representation of Propped Cantilever Beam Modal Amplitude Data

It can be seen in Fig. 1A. to Fig 4A. that modal amplitude data clusters cohesively when there is increment in FE mass. There are 10 classes, i.e. damage location, to classify. For Modal Amplitude data the clustering is happening with respect to 10 damage locations. This is another evidence that predictions are better when there is small amount of extra mass added elastically to the Finite Element.

Appendix B:

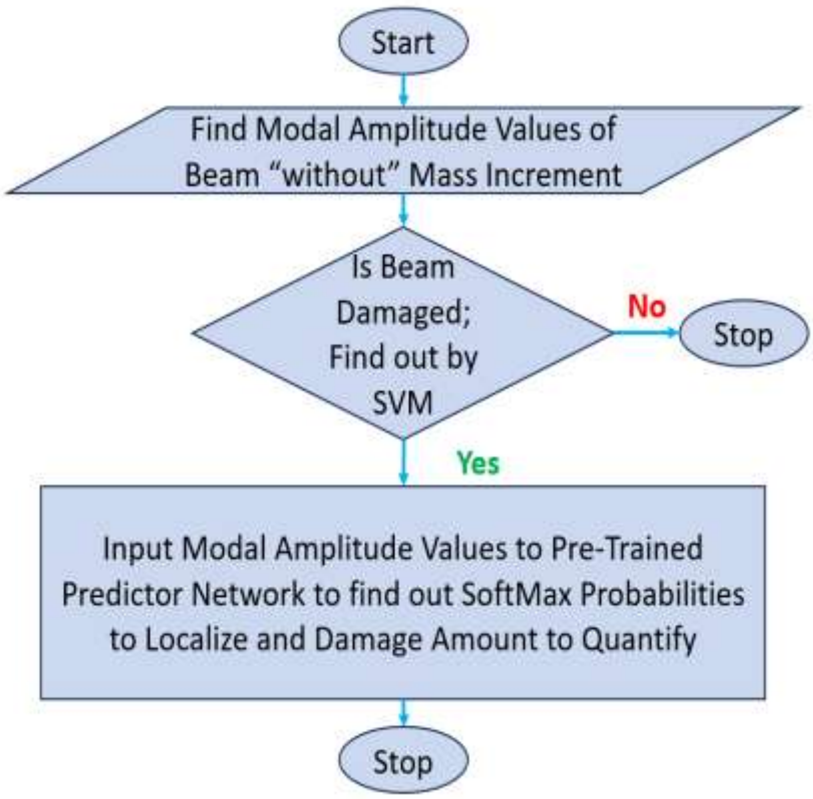

Fig. B1. Flowchart for CBMS

The proposed methodology, as shown in Fig. B1, can be implemented as a Condition-Based Monitoring System (CBMS). Modal amplitude data obtained from the beam without mass increment are used as input to CBMS. A support vector machine is employed to identify the presence of structural damage. Predictor ANN is trained by using extensive amount of modal amplitude data generated through mass increment by FEA simulation. The pre-trained Predictor ANN receives the modal amplitude data from 1-D beam without mass increment. The pre-trained Predictor ANN can localize and quantify the damages from modal amplitude data received.