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Theorems for Arcs of Different Circles Having Same Arc-Angles with Application in Concave and Convex Structures

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Abstract

Arcs are one of the fundamental concepts of circles and are also used in real-life objects such as bridges, jewelleries, buildings, tools etc. In this paper we have proposed theorems for arcs of different circles having the same arc-angles. The proposed theorems have applications in concave/convex surfaces such as lenses and mirrors, buildings, infrastructures, bridges, jewelleries, and mechanical tools such as a sickle.

Keywords: *Geometry, Arc, Circle, Area, Arc length, Bridges, Lens, Mirrors, Tools, Solar panels*

1. Introduction

Arcs are used to build many real-world objects including bridges, tools, jewelleries, buildings, mirrors, lenses etc. The accurate measurement of the curved surfaces using available tools (such as a measuring scale) is error prone. Such as, measurement of arc-length, or measurement of area enclosed by an arc by joining its endpoints via a line segment.

In this paper we have proposed three theorems for arcs of different circles with same arc-angles. These theorems help in accurate calculation of arc lengths, and the area enclosed by an arc and the chord obtained after joining endpoints of the arc.

2. Proposed Theorems

We have proposed three theorems for arcs from different circles with the same arc angle θ (in degree unit) along with their mathematical proofs.

2.1 Arc and Chord Length Theorem

Arcs AB and CD having same arc-angles taken from circles with radius r_1 and r_2 respectively. The ratio of the arc-lengths is equal to the ratio of the line segments obtained after joining the endpoints of the individual arc.

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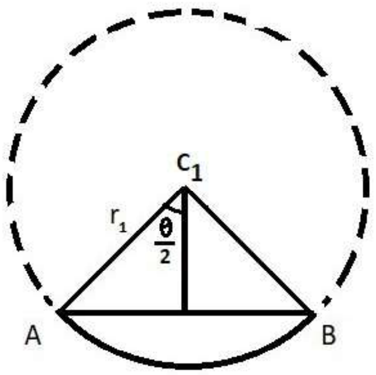

Figure 1: Calculation of Chord Length AB of a Circle with Center at C_1 and Radius r_1

$$\frac{Arc\ Length\ AB}{Arc\ Length\ CD} = \frac{Chord\ Length\ AB}{Chord\ Length\ CD} \quad ...(1)$$

Mathematical Proof

$$Arc\ Length\ AB = 2\pi r_1\left(\frac{\theta}{360}\right) \quad ...(2)$$

Similarly,

$$Arc\ Length\ CD = 2\pi r_2\left(\frac{\theta}{360}\right) \quad ...(3)$$

Using Equation (2) and (3), we get

$$\frac{Arc\ Length\ AB}{Arc\ Length\ CD} = \frac{r_1}{r_2} \quad ...(4)$$

Using Figure 1 for finding the length of chord AB.

$$Chord\ Length\ AB = 2r_1 \sin\left(\frac{\theta}{2}\right) \quad ...(5)$$

Similarly,

$$Chord\ Length\ CD = 2r_2 \sin\left(\frac{\theta}{2}\right) \quad ...(6)$$

Using Equation (5) and (6), we get

$$\frac{Chord\ Length\ AB}{Chord\ Length\ CD} = \frac{r_1}{r_2} \quad ...(7)$$

Now, using Equation (4) and (7)

$$\frac{Arc\ Length\ AB}{Arc\ Length\ CD} = \frac{Chord\ Length\ AB}{Chord\ Length\ CD} \quad ...(8)$$

Hence Proved.

2.2 Arc and Area Theorem

For arc AB and arc CD having same arc-angles taken from circles with radius r_1 and r_2 respectively. The ratio of the areas enclosed after joining the endpoints of the individual arcs (by a line segment) is equal to the square of the ratio of their individual arc-lengths.

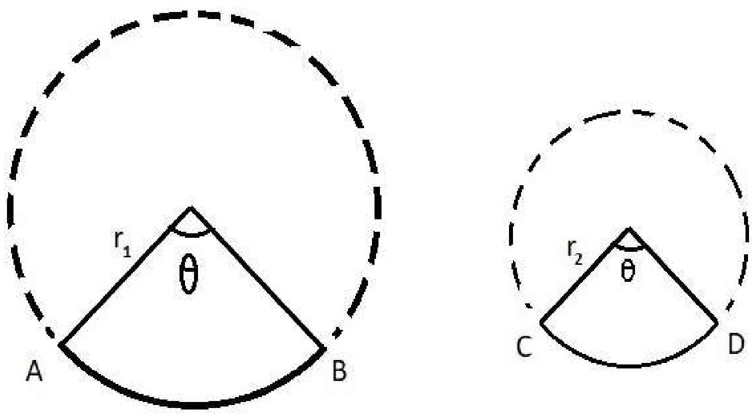

Figure 2: Arcs AB and CD having the Same Arc-Angle in Circle with Radius r_1 and r_2 Respectively

$$\frac{Area\ enclosed\ by\ arc\ AB\ and\ chord\ AB}{Area\ enclosed\ by\ arc\ CD\ and\ chord\ CD}=\left[\frac{Arc\,Length\ AB}{Arc\,Length\ CD}\right]^2 \quad ...(9)$$

Mathematical Proof

Let's assume the arc angle is θ degree as shown in Figure 2.

Area of Sector made with arc $AB=\pi r_1^2\left(\frac{\theta}{360}\right)$ ...(10)

Area of the Triangle $ABC_1=r_1^2\left[\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right]$ ...(11)

Area enclosed by arc AB and chord *AB* = Area of Sector with arc AB - *Area of the Triangle* ABC_1

Now, Using Equation (10) and (11) we can calculate Area enclosed by arc AB and chord AB

$$Area\ enclosed\ by\ arc\ AB\ and\ chord\ AB=r_1^2\left[\pi\left(\frac{\theta}{360}\right)-\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right] \quad ...(12)$$

Similarly,

$$Area\ enclosed\ by\ arc\ CD\ and\ chord\ CD=r_2^2\left[\pi\left(\frac{\theta}{360}\right)-\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right] \quad ...(13)$$

Using Equation (12) and (13), we get

$$\frac{Area\ enclosed\ by\ arc\ AB\ and\ chord\ AB}{Area\ enclosed\ by\ arc\ CD\ and\ chord\ CD}=\left[\frac{r_1}{r_2}\right]^2 \quad ...(14)$$

Replacing the RHS from Equation (4), we get

$$\frac{Area\ enclosed\ by\ arc\ AB\ and\ chord\ AB}{Area\ enclosed\ by\ arc\ CD\ and\ chord\ CD}=\left[\frac{Arc\ Length\ AB}{Arc\ Length\ CD}\right]^2 \quad ...(15)$$

Hence Proved.

2.3 Arc Ratio Theorem

For arc AB and arc CD having same arc-angles taken from circles with radius r_1 and r_2 respectively. The following mathematical expression is always true:

$$\frac{Area\ enclosed\ by\ arc\ AB\ and\ chord\ AB}{Area\ enclosed\ by\ arc\ CD\ and\ chord\ CD} = \left[\frac{Arc\ Length\ AB}{Arc\ Length\ CD}\right]^2 = \left[\frac{Chord\ Length\ AB}{Chord\ Length\ CD}\right]^2 \quad ...(16)$$

Mathematical Proof

Using Arc and Area Theorem represented in Equation (9), we get

$$\frac{Area\ enclosed\ by\ arc\ AB\ and\ chord\ AB}{Area\ enclosed\ by\ arc\ CD\ and\ chord\ CD} = \left[\frac{Arc\ Length\ AB}{Arc\ Length\ CD}\right]^2 \quad ...(17)$$

Using Arc and Chord Length Theorem in Equation (1), we get

$$\frac{Arc\ Length\ AB}{Arc\ Length\ CD} = \frac{Chord\ Length\ AB}{Chord\ Length\ CD} \quad ...(18)$$

Using Equation (17) and (18) we get,

$$\frac{Area\ enclosed\ by\ arc\ AB\ and\ chord\ AB}{Area\ enclosed\ by\ arc\ CD\ and\ chord\ CD} = \left[\frac{Arc\ Length\ AB}{Arc\ Length\ CD}\right]^2 = \left[\frac{Chord\ Length\ AB}{Chord\ Length\ CD}\right]^2$$

Hence Proved.

3. Applications of Arc and Chord Length Theorem in Concave and Convex Surfaces

For two concave mirrors (Zheng *et al.*, 2023) with arcs $A_1E_1B_1$ and $A_2E_2B_2$ having same arc angles created from different size objects, the arc length can be calculated using Arc and Chord Length theorem (Figure 3).

$$\frac{A_2E_2B_2}{A_1E_1B_1} = \frac{A_2B_2}{A_1B_1}$$

$$A_2E_2B_2 = A_1E_1B_1\left(\frac{A_2B_2}{A_1B_1}\right)$$

Similarly for two convex mirrors (Ye *et al.*, 2021) with arcs $C_1F_1D_1$ and $C_2F_2D_2$ having same arc angles created from different size objects, The arc length $C_2F_2D_2$ can be calculated using Arc and Chord Length theorem.

$$C_2F_2D_2 = C_1F_1D_1\left(\frac{C_2D_2}{C_1D_1}\right)$$

These results are also useful in concave, and convex lenses shown in Figure 4.

For two convex lenses (Yammouni and Bruce, 2020) with arcs $A_1E_1B_1$ and $A_2E_2B_2$ having same arc angles created from different size objects, the arc length $A_2E_2B_2$ can be calculated using Arc and Chord Length theorem.

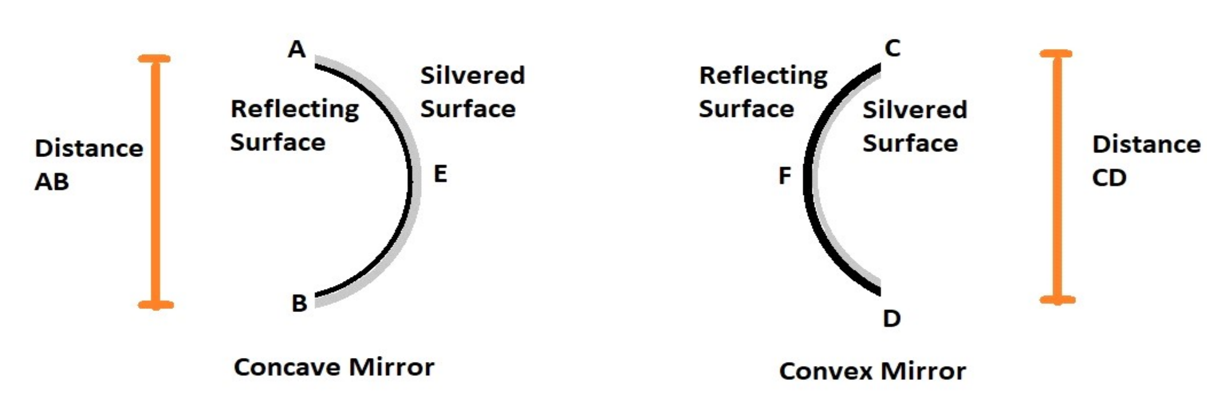

Figure 3: Concave and Convex Mirrors

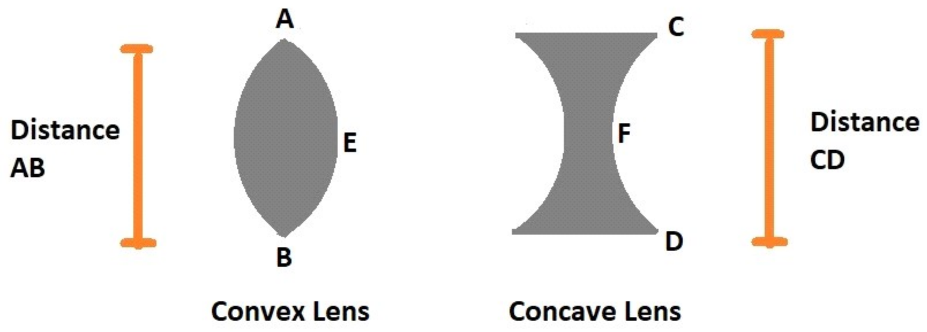

Figure 4: Convex and Concave Lenses

$$A_2E_2B_2 = A_1E_1B_1\left(\frac{A_2B_2}{A_1B_1}\right)$$

Similarly for two concave lenses (Mo *et al.*, 2022) with arcs $C_1F_1D_1$ and $C_2F_2D_2$ having same arc angles created from different size objects, The arc length $C_2F_2D_2$ can be calculated using Arc and Chord Length theorem.

$$C_2F_2D_2 = C_1F_1D_1\left(\frac{C_2D_2}{C_1D_1}\right)$$

Mirrors play a significant part in the field of optics and have a wide usage in developing renewable energy technology such as use of concave, and convex mirrors in solar panels (Siahaan and Hartono, 2019). Along with lenses and mirrors the theorems have significant use cases in Arch Bridges (Gönen and Serdar, 2021; Lan *et al.*, 2020; Sun *et al.*, 2023) when one is trying to construct a bridge as shown in Figure 5 with the help of a simulated miniature model of that bridge as a reference.

4. Applications of Arc and Area Theorem

This theorem has significant usage in construction and cost-estimation of jewelleries, buildings, and infrastructures like-solar panels with concave/convex mirrors (Siahaan and Hartono, 2019), doors, windows, and bridges (Gönen and Serdar, 2021; Lan *et al.*, 2020; Sun *et al.*, 2023) as shown in Figure 5. To apply paint, polish, coating or even cleaning such structures the estimation of area enclosed by arc and chord (chord is created by joining the endpoints of the arc)

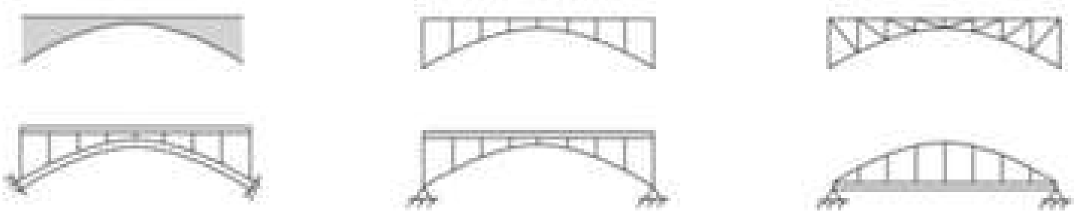

Figure 5: Various Shapes of Bridges

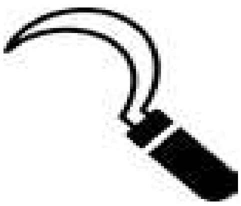

Figure 6: Sickle: A Tool Used in the Agriculture Sector

is required. This theorem is also useful in comparative analysis of mechanical tools such as sickle (Chahal, 2022) that are used in the agriculture sector as shown in Figure 6.

Conclusion

The proposed theorems make it simpler to measure the arc length and area enclosed by arc and chord created by its endpoints given a reference object with the same arc angle. Theorems have significant applications in concave/convex surfaces such as lenses and mirrors; they are also a major part of optics and are useful in solar panels. Theorems also have applications in constructing buildings, infrastructures, bridges, jewelleries, and mechanical tools like sickle where accurate measurement of objects using generally available measuring tools is error prone.

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