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A Fundamental Study of Composite Numbers as a Different Perspective on Problems Related to Prime Numbers

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Abstract

Prime number-related issues can be viewed from drastically different perspectives by examining the close connections between prime numbers and composite numbers. We think that multiple perspectives are the pillars on the path to solutions so we have created this study. As a result of the study, we proposed two new formulas by presenting three theorems and one proof for each theorem, a total of three proofs. We proved that the formula $p \cdot n + p$ returns a composite number in the first of the theorems, which is the preliminary theorem. Our first theorem except the preliminary theorem is that the formula $p \cdot n + p$ returns all composite numbers, and we proved that too. Finally, we created Theorem II using Theorem I to use in our other work and proved that the formula $2 \cdot n \cdot p + p$ returns all odd composite numbers, which is Theorem II. Afterward, we presented the similarities of the $2 \cdot n \cdot p + p$ formula we put forth with another known formula.

Keywords: *Prime numbers, Composite numbers, The prime-composite relationship, The formula of composite numbers, The formula of all composite numbers, The formula of all odd composite numbers*

1. Introduction

Studying prime numbers can be crucial because, among intelligent beings, prime numbers will always have a distinct and unmistakable meaning, regardless of the choice of counting basis and independent of mathematical notation. The riddle surrounding prime numbers, specifically their distribution, has eluded many bright and inquisitive researchers despite their best efforts (Larry, 1973). So, the mystery surrounding the prime numbers continues to grab people's attention.

Composite numbers can be defined as non-prime numbers when evaluated within integers other than 0. So, of course, it is possible to say that there is an impressive interplay between prime numbers and composite numbers and to find incredible correlations between prime numbers and composite numbers.

In this study, basic formulas for composite numbers have been developed so that composite numbers can be used frequently in prime numbers interpretation studies.

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At the beginning of the study, it will be useful to give a few definitions, and more:

Preliminary Information I: Book VII - proposition 30 of Euclid's Elements is the key in the proof of the fundamental theorem of arithmetic (Euclid and Thomas, 1956).

Preliminary Information II: Let x and y be any integer. According to the definition of even numbers, every even number is expressed as $2x$, and according to the definition of odd numbers, every odd number is expressed as $2y + 1$.

Defnition I: Prime numbers are numbers greater than 1 that do not have a factor other than themselves and 1 by the fundamental theorem of arithmetic (Albert , 1990; Atle, 1949).

Def. II: Composite numbers are numbers greater than 1 that do have a factor other than themselves and 1 by the fundamental theorem of arithmetic (Albert , 1990; Aivaras, 2012).

Basis I: Each composite number is expressed as the unique product of more than one prime number by the fundamental theorem of arithmetic and Book VII proposition 31 & Book IX - proposition 14 of Euclid's Elements (Euclid and Thomas, 1956; Göksel and Mehmet, 2001).

Basis II: Every positive integer is either a prime number or a composite number by Definition I and II, Basis I and Book VII - proposition 32 of Euclid's Elements (Euclid and Thomas, 1956).

Notations

$\mathbb{P}$	The set of all prime numbers
$\mathbb{C}$	The set of all composite numbers
$\mathbb{E}$	The set of all even numbers
$\mathbb{O}$	The set of all odd numbers
$\mathbb{N}^+$	The set of all positive natural numbers
$a \in \mathbb{K}$	The element sign: a is an *element* of the set $\mathbb{K}$
$p \wedge q$	The and sign: A conjunction of propositions read as "*p and q*"

Table 1: Truth Table of Conjunction

p	***q***	$p \wedge q$
1	1	1
1	0	0
0	1	0

$p \oplus q$ The xor sign: An exclusive disjunction of propositions read as "*p* or (*xor*) *q*"

Table 2: Truth Table of Exclusive

p	***q***	$p \oplus q$
1	1	0
1	0	1
0	1	1
0	0	0

$p \Rightarrow q$ The if sign: A conditional statement read as "*if p*, then *q*"

Table 3: Truth Table of Implication

p	***q***	$p \Rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

$p \Leftrightarrow q$ The if and only if sign: A biconditional statement read as "*p if and only if q*"

Table 4: Truth Table of Biconditional		
p	q	$p \Leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

$\{x \in \mathrm{P} \mid x^2\}$ The given that sign: This set consists of x^2 values, *where* x is a prime number

$a \mid b$ The divides sign: *a divides b* without a remainder

$\therefore a \in \mathbb{K}$ The therefore sign: *Therefore*, a is an element of the set $\mathbb{K}$

$a \mathrm{R} b \Leftrightarrow a \mid b$ The relation sign: *a related* with b if and only if a divides b

$\forall a(a+1>0)$ The universal quantifier: *For every a number*, $a + 1$ is greater than 2

$a_b \in \mathbb{K}$ The sequence sign: A *sequence* of numbers from the set $\mathbb{K}$, $a_1 = x$, $a_2 = y$, ..., $a_b = z$

2. Theorems and Proofs

Preliminary Theorem: For all distinct prime numbers p and for all distinct positive natural numbers n, then $p \cdot n + p$ is a composite number.

$p \in \mathbb{P}$

$n \in \mathbb{N}^+$

$(\forall p)(\forall n)(p \cdot n + p) \in \mathbb{C}$

Preliminary Proof: The expression $p \cdot n + p$ can be written as $p \cdot (n + 1)$ by distributive property[1].

$p \cdot n + p = p \cdot (n + 1)$ by distributive property

If n is any positive natural number and 1 is a positive natural number, then $n + 1$ equals to be a positive natural number by the closure property of positive natural numbers under addition[2].

$(n \in \mathbb{N}^+ \Lambda\ 1 \in \mathbb{N}^+) \Rightarrow (n + 1 \in \mathbb{N}^+)$ by closure property of positive natural numbers under addition

If n is any positive natural number, then $n + 1$ equals to be either the composite number or the prime number, depending on Basis II.

$(n+1 \in \mathbb{N}^+) \Rightarrow ((n+1 \in \mathbb{P}) \oplus (n+1 \in \mathbb{C}))$ by Basis II

Let q_x be a prime number and c_x be a composite number. If $n + 1$ is a prime number, then $p \cdot (n + 1)$ equals to be $p \cdot q_x$. If $n + 1$ is a composite number, then $p \cdot (n + 1)$ equals to be $p \cdot c_x$.

$q_x \in \mathbb{P}$

$c_x \in \mathbb{C}$

$(n+1 \in \mathbb{P}) \Rightarrow (p \cdot (n+1) = p \cdot q_x) \oplus (n+1 \in \mathbb{C}) \Rightarrow (p \cdot (n+1) = p \cdot c_x)$

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1 The distributive property states that A · (B ± C) can be expressed such that A · (B ± C) = A · B ± A·C, and both operations will give the same result. This means that the value A to be multiplied is distributed separately to the other two values and then added or subtracted with each other in the order in which they are given. As can be seen in the definition, it has the property of distribution according to multiplication, addition, and subtraction.

2 The closure property is defined as the closure of a set of numbers using arithmetic operations. In other words, if the result obtained when an operation is performed on any two numbers from the set is always a number from that set, that set of numbers is closed. Natural numbers have the closure property under addition and multiplication. If two natural numbers are added or multiplied, then the result equals always a natural number. Natural numbers do not have the closure property under subtraction and division. If two natural numbers are subtracted or divided, then the result is equal to not always a natural number. Based on the fact that natural numbers have the closure property under addition, It can be said that two positive natural numbers have the closure property under addition because the set of positive natural numbers is a subset of the set of natural numbers. This is true for any positive natural numbers pair. To sum up, every pair of positive natural numbers have the closure property under addition.

The result of the expression $p \cdot q_x$ has divisors p and q_x besides itself and 1. So the expression $p \cdot q_x$ is a composite number according to Definition II. The result of the expression $p \cdot c_x$ has divisors p and c_x besides itself and 1. So the expression $p \cdot c_x$ is a composite number according to Definition II. Therefore, for all distinct prime numbers p and for all distinct positive natural numbers n, then $p \cdot (n+1)$, that is, $p \cdot n + p$ equals a composite number.

$$a \mathrm{R}\, p \cdot q_x \Leftrightarrow a \mid p \cdot q_x$$

$$(a = q_x) \Rightarrow \mathrm{R} = \{1, p \cdot q_x, p, q_x\}$$

$$(a = c_x) \Rightarrow \mathrm{R} = \{1, p \cdot c_x, p, c_x\}$$

$$\therefore (\forall p)\,(\forall n)\,(p \cdot (n+1) = p \cdot n + p) \in \mathbb{C} \text{ by Def. II}$$

Theorem I: Let p be a prime number and n be a positive natural number. Then the formula $p \cdot n + p$ returns all composite numbers.

$$\mathbb{C} = \{p \in \mathbb{P}, n \in \mathbb{N}^+ \mid p \cdot n + p\}$$

Proof I: As a result of the work on the proof of the preliminary theorem, the following can be said: with p prime, $n + 1$ either prime or composite, this operation can be expressed as: "*prime* × (either *prime* or *composite*)". So there are two possibilities:

Possibility I: If $n + 1$ is a composite number, then $p \cdot (n + 1)$ can be expressed as "*prime* × *composite*". Adhering to Basis I, this expression can be written as "*prime* × (*prime* × ⋯ × *prime*)".

$$c \in \mathbb{C}$$

$$(n+1 \in \mathbb{C}) \Rightarrow (p \cdot (n+1) = p \cdot c)$$

sec.[3] Basis I:

$$j, k \in \mathbb{N}^+$$

$$t_j, q_k \in \mathbb{P}$$

$$a = t_1 \cdot t_2 \cdot \cdots \cdot t_j$$

$$b = q_1 \cdot q_2 \cdot \cdots \cdot q_k$$

$$c = a \cdot b$$

$$\therefore p \cdot c = p \cdot a \cdot b = p \cdot (t_1 \cdot t_2 \cdot \cdots \cdot t_j) \cdot (q_1 \cdot q_2 \cdot \cdots \cdot q_k)$$

Possibility II: If $n + 1$ is a prime number, then $p \cdot (n + 1)$ can be expressed as "*prime* × (*prime*)".

$$x \in \mathbb{N}^+$$

$$p_x \in \mathbb{P}$$

$$(n+1 \in \mathbb{P}) \Rightarrow (p \cdot (n+1) = p \cdot p_x)$$

As a result, there are two potential outcomes, "*prime* × (*prime* × · · · × *prime*)" and "*prime* × (*prime*)". Since these two results are potentially the product of all prime numbers; so it can be said, adhering to Basis I, that this formula $p \cdot n + p$ returns all composite numbers.

$$(n+1 \in \mathbb{C}) \Rightarrow (p \cdot (n+1) = p \cdot (t_1 \cdot t_2 \cdot \cdots \cdot t_j) \cdot (q_1 \cdot q_2 \cdot \cdots \cdot q_k)) \oplus (n+1 \in \mathbb{P})$$

$$\Rightarrow (p \cdot (n+1) = p \cdot p_x)$$

sec. Basis I:

$$\therefore \mathbb{C} = \{p \in \mathbb{P}, n \in \mathbb{N}^+ \mid p \cdot n + p\}$$

Since there are no even prime numbers except for 2, the following theorem can be established to obtain only odd composite numbers:

[3] The abbreviation "sec." comes from the Latin word "secundum" and it denotes "following" or "in accordance with". It is used in the context of "sec. Mustafa Kemal Atatürk", which means "according to Mustafa Kemal Atatürk".

Theorem II: Let p be a prime number and n be a positive natural number. Then the formula $2 \cdot n \cdot p + p$ returns all odd composite numbers.

Proof II: The formula $p \cdot n + p$ obtained in **Theorem I** has four possibilities arising from the fact that p can be *even*/*odd* and n can be *even*/*odd* :

Table 5: Table of All Possibilities

Options	p	n	$p \cdot (n+1)$
Option I	*even*	*even*	$even \cdot (even + 1)$
Option II	*even*	*odd*	$even \cdot (odd + 1)$
Option III	*odd*	*even*	$odd \cdot (even + 1)$
Option IV	*odd*	*odd*	$odd \cdot (odd + 1)$

Option I: Let k, t, z, and m be positive natural numbers. If p is an even number and n is an even number, then let p and n be equal to $2k$ and $2t$, respectively, according to the definition of even numbers mentioned in Preliminary Information II. If p is $2k$ and n is $2t$, then $p \cdot (n+1)$ equals to be $2k \cdot (2t+1)$. Let $2t+1$ be equal to z. If $2t+1$ is z, then $2k \cdot (2t+1)$ equals to be $2kz$. Let kz be equal to m. If k is m, then $2kz$ equals to be $2m$, which is an even number by the definition of even numbers. Therefore, if p is $2k$ and n is $2t$, then $p \cdot (n+1)$ is an even number.

$k, t, z, m \in \mathbb{N}^+$

$(p \in \mathbb{E}) \Rightarrow (p = 2k)$ by def. of even numbers

$(n \in \mathbb{E}) \Rightarrow (n = 2t)$ by def. of even numbers

$((p = 2k) \Lambda (n = 2t)) \Rightarrow (p \cdot (n+1) = 2k \cdot (2t+1))$

$(2t+1 = z) \Rightarrow (2k \cdot (2t+1) = 2kz)$

$(kz = m) \Rightarrow (2kz = 2m \in \mathbb{E})$ by def. of even numbers

$\therefore ((p = 2k) \Lambda (n = 2t)) \Rightarrow (p \cdot (n+1) \in \mathbb{E})$

Option II: If p is an even number, then let p be equal to $2k$, and if n is an odd number, then let n be equal to $2t + 1$ according to the definition of even numbers and odd numbers mentioned in Preliminary Information II. If p is $2k$ and n is $2t+1$, then $p \cdot (n+1)$ equals to be $2k \cdot (2t+1+1)$ equals to be $2k \cdot (2t+2)$. Let $2t+2$ be equal to z. If $2t+2$ is z, then $2k \cdot (2t+2)$ equals to be $2kz$. Let kz be equal to m. If kz is m, then $2kz$ equals to be $2m$, which is an even number by the definition of even numbers. Therefore, if p is $2k$ and n is $2t+1$, then $p \cdot (n+1)$ is an even number.

$(p \in \mathbb{E}) \Rightarrow (p = 2k)$ by def. of even numbers

$(n \in \mathbb{O}) \Rightarrow (n = 2t+1)$ by def. of odd numbers

$((p = 2k) \Lambda (n = 2t+1)) \Rightarrow (p \cdot (n+1) = 2k \cdot (2t+1+1) = 2k \cdot (2t+2))$

$(2t+2 = z) \Rightarrow (2k \cdot (2t+2) = 2kz)$

$(kz = m) \Rightarrow (2kz = 2m \in \mathbb{E})$ by def. of even numbers

$\therefore ((p = 2k) \Lambda (n = 2t+1)) \Rightarrow (p \cdot (n+1) \in \mathbb{E})$

Option III: If p is an odd number, then let p be equal to $2k+1$, and if n is an even number, then let n be equal to $2t$ according to the definition of odd numbers and even numbers mentioned in Preliminary Information II. If p is $2k+1$ and n is $2t$, then $p \cdot (n+1)$ equals to be $(2k+1) \cdot (2t+1)$ equals to be $4kt + 2k + 2t + 1$ equals to be $2 \cdot (2kt) + 2 \cdot (k + t) + 1$. Let $2kt$ and $k+t$ be equal to z and m, respectively. If $2kt$ is z and $k+t$ is m, then $2 \cdot (2kt) + 2 \cdot (k+t) + 1$ equals to be $2z + 2m + 1$ equals to be $2 \cdot (z+m) + 1$. Let $z+m$ be equal to a. If $z+m$ is a, then $2 \cdot (z+m) + 1$ equals to be $2a + 1$, which is an odd number by the definition of odd numbers. Therefore, if p is $2k+1$ and n is $2t$, then $p \cdot (n+1)$ is an odd number.

$a \in \mathbb{N}^+$

$(p \in \mathbb{O}) \Rightarrow (p = 2k+1)$ by def. of odd numbers

$(n \in \mathbb{E}) \Rightarrow (n = 2t)$ by def. of even numbers

$((p = 2k + 1) \land (n = 2t))$

$\Rightarrow (p \cdot (n + 1) = (2k + 1) \cdot (2t + 1) = 4kt + 2k + 2t + 1 = 2 \cdot (2kt) + 2 \cdot (k + t) + 1)$

$((2kt = z) \land (k + t = m)) \Rightarrow (2 \cdot (2kt) + 2 \cdot (k + t) + 1 = 2z + 2m + 1 = 2 \cdot (z + m) + 1)$

$(z + m = a) \Rightarrow (2 \cdot (z + m) = 2a + 1 \in \mathbb{O})$ by def. of odd numbers

$\therefore ((p = 2k + 1) \land (n = 2t)) \Rightarrow (p \cdot (n + 1) \in \mathbb{O})$

Option IV: Let a be a positive natural number. If p is an odd number and n is an odd number, then let p and n be equal to $2k + 1$ and $2t + 1$, respectively, according to the definition of odd numbers mentioned in Preliminary Information II. If p is $2k + 1$ and n is $2t + 1$, then $p \cdot (n + 1)$ equals to be $(2k + 1) \cdot (2t + 1 + 1)$ equals to be $(2k + 1) \cdot (2t + 2)$ equals to be $4kt + 4k + 2t + 2$ equals to be $2 \cdot (2kt) + 2 \cdot (2k) + 2t + 2$. Let $2kt$ and $2k$ be equal to z and m, respectively. If $2kt$ is z and $2k$ is m, then $2 \cdot (2kt) + 2 \cdot (2k) + 2t + 2$ equals to be $2z + 2m + 2t + 2$ equals to be $2 \cdot (z + m + t + 1)$. Let $z + m + t + 1$ be equal to a. If $z + m + t + 1$ is a, then $2 \cdot (z + m + t + 1)$ equals to be $2a$, which is an even number by the definition of even numbers. Therefore, if p is $2k + 1$ and n is $2t + 1$, then $p \cdot (n + 1)$ is an even number.

$(p \in \mathbb{O}) \Rightarrow (p = 2k + 1)$ by def. of odd numbers

$(n \in \mathbb{O}) \Rightarrow (n = 2t + 1)$ by def. of odd numbers

$((p = 2k + 1) \land (n = 2t + 1))$

$\Rightarrow (p \cdot (n + 1) = (2k + 1) \cdot (2t + 1 + 1) = (2k + 1) \cdot (2t + 2) = 4kt + 4k + 2t + 2 = (2kt) + 2 \cdot (2k) + 2t + 2)$

$((2kt = z) \land (2k = m)) \Rightarrow (2 \cdot (2kt) + 2 \cdot (2k) + 2t + 2 = 2z + 2m + 2t = 2 \cdot (z + m + t + 1))$

$(z + m + t + 1 = a) \Rightarrow (2 \cdot (z + m + t + 1) = 2a \in \mathbb{E})$ by def. of even numbers

$\therefore ((p = 2k + 1) \land (n = 2t + 1)) \Rightarrow (p \cdot (n + 1) \in \mathbb{E})$

As a result, a table like this can be created:

Table 6: Table of Results of All Possibilities

Options	***p***	***n***	***p · (n + 1)***
Option I	*even*	*even*	*even*
Option II	*even*	*odd*	*even*
Option III	*odd*	*even*	*odd*
Option IV	*odd*	*odd*	*even*

The third option is the only way to return an odd number from the formula that gives all composite numbers. In this case, a formula that satisfies the requirements of the third option returns all odd composite numbers. Because the third option is derived from the formula that gives all composite numbers, and a result is always an odd number. The requirements of the third option are that the prime number p is an odd prime number and the positive natural number n is a positive even natural number in the formula $p \cdot (n + 1)$. In this context, the number $n + 1$ is also odd. Let p be an odd number to generate the formula that returns only odd composite numbers. Provided that p is constant, one composite number must be obtained for each natural number. That's why we need to manipulate the $n + 1$ value in the formula $p \cdot (n + 1)$ so that it is always an odd number. So if we multiply n by 2 for satisfying the requirements of the third option, we get $p \cdot (2n + 1)$, where $2n + 1$ is an odd number according to the definition of odd numbers in Preliminary Information II. For all distinct prime numbers p, as proved above, the formula $p \cdot (n + 1)$ returns all odd composite numbers as long as n is any positive even natural number; if we want the formula to return all odd composite numbers for each positive natural number, that is, to be more optimized, the formula $p \cdot (2n + 1)$ derived from the formula $p \cdot (n + 1)$ gives all odd composite numbers as long as n is any positive natural numbers.

3. Results

For the potential of the relationship between composite numbers and prime numbers to provide solutions to mathematical problems related to prime numbers, we have presented three theorems, one of which is a preliminary theorem, and one proof for each theorem, a total of three proofs.

As a result, we proved that formula $n \cdot p + p$ returns all composite numbers, and the formula $2 \cdot n \cdot p + p$ returns all odd composite numbers.

Since we have explained these similarities and differences in the Appendix.

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Appendix

By far, the most familiar and most optimized formula for returns odd composite numbers is the $2 \cdot n \cdot p + p^2$ formula developed by Marouane Rhafli. This formula returns all odd composite numbers where p is any odd prime number and n is any natural number (Marouane Rhafli, 2019).

The $2 \cdot n \cdot p + p$ formula we developed returns all composite numbers where p is any odd prime number and n is any positive natural number.

The formula we developed will be used in our next study to offer another perspective on the **twin prime conjecture**.

Finally, we can attribute the similarity of the two formulas developed to the waggish of mathematics.

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