https://doi.org/10.51483/IJPAMR.2.2.2022.63-74 ISSN: 2789-9160

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The Hall π-Subgroups of Some of the Classical Simple Groups

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Article Info

Volume 2, Issue 2, October 2022
Received : 12 July 2022
Accepted : 22 September 2022
Published : 05 October 2022
doi: 10.51483/IJPAMR.2.2.2022.63-74

Abstract

The aim of this work is using the information in the ATLAS of Finite Groups (Wilson *et al.*, 1985) and by developing a program inside the GAP computational system (The GAP computational System , 2010), to determine all Hall π-subgroups for some finite classical simple groups such as some of the finite unitary and finite simplectic simple groups and some of finite simple groups of of Lie type. The structures and permutation representations of the Hall π-subgroups have been found. By using the following theoretical and computational algorithms, we determined the solvable subgroups of large order of the finite non-abelian simple linear groups $G = L_2(p) = PSL(2, p)$, for $p \geq 5$ and p is a prime number, also their presentations and permutation representations have been found.

Keywords: *Maximal subgroup, Solvable, p-nilpotent, Formation*

1. Approach of Collecting Hall Subgroups

We combine the information in the ATLAS (Wilson *et al.*, 1985) with explicit computations using the GAP system (The GAP computational System , 2010), in particular its library of finite groups and its Character Table Library. We can define the non-abelian finite simple groups, by using the GAP system, as follows:

1.1. Cases Where the Structure of the Finite Simple Group is Available in the GAP Library

The non-abelian finite simple groups who's Structures are available in the GAP library are collected by using the following GAP-functions:

The only Symplectic classical groups of dimension n over the fields GF(q) which appear in the ATLAS (Wilson *et al.*, 1985), are:

$S_4(2)'$ $S_4(3)$ $S_4(4)$ $S_4(5)$ $S_4(7)$ $S_4(9)$ $S_4(11)$ $S_4(13)$ $S_4(17)$ $S_6(2)$ $S_6(3)$ $S_8(2)$ $S_8(3)$ $S_{10}(2)$

By using the following command:

gap> g:=SymplecticGroup(n,q); or

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gap> g:=PSP(n,q);

The only Unitary classical groups of dimension n over the fields GF(q) which which appear in the ATLAS (Wilson *et al.*, 1985), are:

$U_3(3)$ $U_3(4)$ $U_3(5)$ $U_3(7)$ $U_3(8)$ $U_3(9)$ $U_3(11)$ $U_3(13)$ $U_4(2)$ $U_4(3)$ $U_4(4)$ $U_4(5)$ $U_5(2)$ $U_5(3)$ $U_5(4)$ $U_6(2)$ $U_6(3)$ $U_7(2)$

By using the following command:

GAP> g:=ProjectiveSpecialUnitaryGroup(n,q); or

GAP> g:=PSU(n,q)

The only simple groups of Lie type which can be directly collected from the GAP-command which also appear in ATLAS (Wilson *et al.*, 1985), are:

R(27), Sz(8), and Sz(32)

By using the following command:

GAP> g:=Suz(8); or gap> g:=SuzukiGroup(8);

GAP> g:=Suz(32); or gap> g:=SuzukiGroup(32);

GAP> g:=Ree(27); or gap> g:=ReeGroup(27);

1.2. Cases Where the Structure of the Finite Simple Group is Not Available in the GAP Library

The non-abelian finite simple groups who's Structures are not available in the GAP library can be collected by using their generators appear in the ATLAS.

2. Calculating Hall π-Subgroups

We develop the following program, by intensive uses of some of the GAP functions, to compute the orders of the Hall π-subgroup *M* in the finite non-abelian simple group *G* which appeared in the Atlas of Finite Groups (Wilson *et al.*, 1985). Also this program by using the information in the Atlas and the GAP system, in particular the Character Table Library (Breuer, 2012), finds the structure of *M*, its presentations with generators, its conjugacy classes of elements with their fusions map in *G*, and also its permutation representations in *G*. Also this program investigate some properties of *M*.

```
s:=Set(Factors(Size(g)));;        "find the set of primes of order of the simple group g"
q:=PartitionsSet(s,2);;           "Find the all proper subsets of s"
Display("--------------------------------");
for j in [1 .. Length(q)] do
> r:=q[j];
> for i in [1 .. 2] do
> if Length(r[i])>1 then
hhh:=HallSubgroup(g,r[i]);
if hhh= fail then
> Print("? = ");Print(r[i]);Display("    ");
Display("No Hall ?-Subgroup");Display("---------------");
elif hhh<> fail then
Display("   ");
Print("the Hall");Print(r[i]);Print("-subgroup H of G is the  ");
```

```
Display(hhh);
order:=Size(hhh);Display("  "); Print(" and its oreder is  ");Print(order);Display("   ");
gen:=SmallGeneratingSet(hhh);
Display(" and H is generated by  ");Display(gen);
Display(" and H is Isomorphic to the group  ");
struc:=StructureDescription(hhh);
Display(struc);
kkkk:=Group(gen);
hhh:=kkkk;
cc:=CharacterTable(hhh);;
Display(cc);
cname:=ClassNames(c);;
ccname:=ClassNames(cc);;
fuss:=FusionConjugacyClasses(cc,c);
fus1:=fuss;
Display("The Fusion Maps are  ");
for rw in [1 .. Number(fuss)] do
 tw:=fuss[rw];;
 qw:=cname[tw];;
 Print(qw);Print("  ");
  od;
 Print("                    ");
 Display("                    ");
Display("The Induced Character is = ");
ind:=PermChars(c,Size(c)/Size(cc));
perm:=PermCharInfo(c,ind) .ATLAS;
Display(perm);
Display("-----------------------------");
Display("Some properties of this Hall Subgroup : ");
 z1:=IsSimple(hhh);;
 z2:=IsAbelian(hhh);;
 z3:=IsNilpotent(hhh);
 z4:=IsSupersolvable(hhh);;
z5:=IsSolvable(hhh);
z6:=IsCyclic(hhh);
> fi;
fi;od;
```

Display("---------------------------------");

> od;

> od;

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Appendix 1

Finite Simple Group G	The Hall π-Subgroup M of G (These Results Obtained by Applying the GAP - Procedures Appear in Chapter (2))		
	E_π Satisfied When π =	Structure and Properties of M	Representations
G = The Exceptional group G2(2)'	**G2(2)' has no Hall π-subgroups**		
$\lvert G\rvert = 2^5.3^3.7$			
$G = \langle a, b : a^2 = b^6 = (ab)^7 = 1\rangle$			
G2(2)' = <a = bin1, b = bin1>			
bin1 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/clas/U33/gap/U33G1-p28B0.g1 .			
and bin1 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/clas/U33/gap/U33G1-p28B0.g2 .			
G = The Exceptional group G2(4)	**G2(4) has no Hall π-subgroups**		
$\lvert G\rvert = 2^{12}.3^3.5^2.7.13$			
$G = \langle a, b : a^2 = b^5 = (ab)^{13} = (abb)^{13} = 1 \rangle$			
G2(4) = <a = b11, b = b21>			
b11 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/exc/G24/gap/G24G1-p416B0.g1 .			
and b21 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/exc/G24/gap/G24G1-p416B0.g2 .			
G = The Exceptional group ^{3}D4(2)	**^{3}D4(2) has no Hall π-subgroups**		
$\lvert G\rvert = 2^{12}.3^4.7^2.13$			
$G = \langle a, b: a^2 = b^9 = (ab)^{13} = (abb)^8 = 1 \rangle$			
^{3}D4(2) = <a = b11, b = b21>			
b11 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/exc/TD42/gap/TD42G1-p819B0.g1 .			
and b21 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/exc/TD42/gap/TD42G1-p819B0.g2 .			

Appendix 2

Finite Simple Group G	The Hall π-Subgroup M of G (These Results Obtained by Applying the GAP - Procedures Appear in Chapter (2))		
	E_π Satisfied When π =	**Structure and Properties of M**	**Representations**
G = The Suzuki group Sz(8)	{2,7}	The Hall {2,7} subgroup Sz(8) of M is of order 448 . M can be generated by :M=< [[[Z(2^3)^3, 0*Z(2), 0*Z(2), 0*Z(2)], [0*Z(2), Z(2^3)^2, 0*Z(2), 0*Z(2)], [0*Z(2), 0*Z(2), Z(2^3)^5, 0*Z(2)],[0*Z(2), 0*Z(2), 0*Z(2), Z(2^3)^4]],[[Z(2)^0, 0*Z(2), 0*Z(2), 0*Z(2)], [Z(2^3)^5, Z(2)^0, 0*Z(2), 0*Z(2)], [Z(2^3), Z(2^3)^6, Z(2)^0, 0*Z(2)],[Z(2^3)^5, Z(2^3)^2, Z(2^3)^5, Z(2)^0]]]>	The Fusion Maps of the conjugacy classes of M into G are:
$\lvert G\rvert = 2^6.5.7.13$		and it is Isomorphic to the group $((C_2 \times C_2 \times C_2) . (C_2 \times C_2 \times C_2) . C_7$	1a 2a 4a 4b 7c 7b 7a 7a 7b 7c .
$G = \langle a, b : a^2 = b^4 = (ab)^5 = (abb)^7 = (abab^3ab^2)^7 = 1 \rangle$		Some properties of M:	
Sz(8) = <a = b11, b = b21>		M is SIMPLE : false	The Induced Character of $1_M \uparrow^G$ **is = 1a + 64a**
b11 can be obtained from:		M is ABELIAN : false	
http://brauer.maths.qmul.ac.uk/Atlas/exc/Sz8/gap/Sz8G1-p65B0.g1 .		M is Cyclic : false	
and b21 can be obtained from:		M is NILPOTENT : false	
http://brauer.maths.qmul.ac.uk/Atlas/exc/Sz8/gap/Sz8G1-p65B0.g2 .		M is Solvable : true	
G = The Exceptional group $^2F4(2)'$	**$^2F4(2)'$ has no Hall π-subgroups**		
$\lvert G\rvert = 2^{11}.3^3.5^2.13$			
$G = \langle a, b : a^2 = b^3 = (ab)^{13} = 1 \rangle$.			
$^2F4(2)'$ = <a = b11, b = b21>			
b11 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/exc/TF42/gap/TF42G1-p1600B0.g1			
and b21 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/exc/TF42/gap/TF42G1-p1600B0.g2			
G = The Symplectic group group $S_4(2)'$	**$S_4(2)'$ has no Hall π-subgroups**		
$\lvert G\rvert = 2^3.3^2.5$			
$G = \langle a, b : a^2 = b^4 = (ab)^5 = 1 \rangle$			
z1 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/alt/A6/gap/A6G1-p6aB0.g1 .			
and z1 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/alt/A6/gap/A6G1-p6aB0.g2 .			

Appendix 3

Finite Simple Group G	The Hall π-Subgroup M of G (These Results Obtained by Applying the GAP – Procedures Appear in Chapter (2))		
	E_π Satisfied When π =	Structure and Properties of M	Representations
G = The Symplectic group $S_4(3)$	**$S_4(3)$ has no Hall π-subgroups**		
$\|G\| = 2^6.3^4.5$			
$G = \langle a, b : a^2 = b^5 = (ab)^9 = 1 \rangle$			
$S_4(3)$ = <a = bin1, b = bin1>			
bin1 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/clas/U42/gap/U42G1-p27B0.g1 .			
and bin1 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/clas/U42/gap/U42G1-p27B0.g2 .			
G = The Symplectic group $S_4(4)$	{2,3}	The Hall {2,3}-subgroup M of $S_4(4)$ is of order 2304. M can be generated by: M = < (2,17,68,7,79,48)(3,62,38,24,21,44)(4,25,36,16,61,71)(5,70)(6,15,46,30,77,27)(8,64,82,53,26,22)(9,47,75,40,50,45)(10,58,31,33,67,54)(11,60,85)(12,80)(13,23,69,35,41,20)(14,28,73,83,59,74)(18,65,42,76,51,57)(19,72,55,52,78,49)(29,39,34,81,63,56)(32,84)(43,66)(2,46,33,48,16,44)(3,71,81,13,29,36)(4,34,50,27,30,40)(5,20,78,79,52,67)(6,68,23,56,7,31)(8,22,66,65,76,43)(9,41,38,47,54,10)(11,37)(12,53,18,80,51,82)(14,58,84,61,72,45)(15,70,25,28,17,83)(19,77,32,69,59,75)(21,55,63,49,62,73)(24,35)(26,57,64)(39,74)(60,85) >,	The Fusion Maps of the conjugacy classes of M into G are: 1a, 2c, 2a, 2b, 4b, 4a, 2a, 2c, 2c, 2b, 3b, 6a, 3b, 6a, 3b, 6a, 3a, 6b, 3a, 6b, 3b, 6a, 3a, 6b, 3a, 6b.
$\|G\| = 2^8.3^2.5^2.17$		and it is Isomorphic to the group $((C_2 \times C_2 \times ((C_2 \times C_2 \times C_2) : C_2)) : C_2) : (C_3 \times C_3)$.	The Induced Character of $1_M \uparrow^G$ is = 1a + 34ab + 50aa + 256a
$G = \langle a, b : a^2 = b^5 = (ab)^{17} = (ababb)^{15} = 1 \rangle$		Some properties of M:	
$S_4(4)$ = <a = bin1, b = bin1>		M is SIMPLE : false	
bin1 can be obtained from:		M is ABELIAN : false	
http://brauer.maths.qmul.ac.uk/Atlas/clas/S44/gap/S44G1-p85aB0.g1 .		M is Cyclic : false	
and bin1 can be obtained from:		M is NILPOTENT : false	
http://brauer.maths.qmul.ac.uk/Atlas/clas/S44/gap/S44G1-p85aB0.g2 .		M is Solvable : true	

Appendix 4

Finite Simple Group G	The Hall π-Subgroup M of G (These Results Obtained by Applying the GAP – Procedures Appear in Chapter (2))		
	E_π Satisfied When π =	Structure and Properties of M	Representations
G =The Symplectic group $S_4(5)$	{2,3}	The Hall {2,3}-subgroup M os $S_4(5)$ is of order 576. M can be generated by: M = < (1,144,31,150,10,123)(2,87,28,72,132,53)(3,115,118,63,32,66)(4,83,151,17,121,95)(5,94,112)(6,13,134,89,126,16)(7,111,15,11,133,141)(8,59,108)(9,143,43,71,127,97)(12,107,81,145,8,30)(14,76,101,125,50,37)(18,57,48,22,74,70)(19,68,119,29,21,140)(20,154,135,90,122,88)(23,65,36,79,138,98)(24,82,103,128,39,86)(25,155,117,38,153,136)(26,27,52,44,113,99)(33,102,109,69,40,67)(34,61,45,49,73,93)(35,92,129,91,114,96)(41,54,104)(42,105,77,152,55,148)(46,62,147)(47,78,58,84,64,130)(51,75,110,60,137,131)(56,149,156,116,120,100)(80,106,12,139,146,142),(1,69,102)(2,154,19)(3,42,86)(4,92,135)(5,71,9)(6,82,65)(7,38,99)(8,100,156)(10,31,67)(11,52,25)(12,148,91)(13,68,85)(14,46,125)(15,136,121)(16,61,50)(17,88,114)(18,147,22)(20,108,90)(21,126,107)(23,24,64)(26,116,137)(27,115,124)(28,149,43)(29,122,72)(3,81,62)(32,113,142)(33,150,40)(34,94,49)(35,77,145)(36,146,87)(37,45,130)(39,89,138)(41,60,51)(44,75,56)(47,143,74)(48,105,131)(53,97,120)(54,133,111)(55,70,110)(57,127,84)(58,93,101)(59,153,155)(63,103,152)(66,80,151)(73,134,76)(78,128,79)(83,117,141)(95,139,118)(96,129,104)(98,132,106)(109,123,144)(112,140,119) >,	The Fusion Maps of the conjugacy classes of M into G are: 1a, 2b, 2a, 4b, 3a, 6c, 3a, 6c, 3a, 6c, 12b, 6b, 3b, 12b, 6b, 3b, 2b, 2a, 4a, 6c, 6a, 6c, 6a.
$\|G\| = 2^6.3^2.5^4.13$		and it is Isomorphic to the group $(((C_2 \times C_2 \times C_2) : (C_2 \times C_2)) : (C_3 \times C_3)) : C_2$.	The Induced Character of $1_M \uparrow^G$ is =
$G = \langle a, b : a^2 = b^3 = (ab)^{13} = 1 \rangle$		Some properties of M:	
$S_4(5)$ = <a = bin1, b = bin1>		M is SIMPLE : false	
bin1 can be obtained from:		M is ABELIAN : false	
http://brauer.maths.qmul.ac.uk/Atlas/clas/S45/gap/S45G1-p156aB0.g1 .		M is Cyclic : false	
and bin1 can be obtained from:		M is NILPOTENT : false	
http://brauer.maths.qmul.ac.uk/Atlas/clas/S45/gap/S45G1-p156aB0.g2 .		M is Solvable : true	
G = The Symplectic group $S_4(9)$	**$S_4(9)$ has no Hall π-subgroups**		
$\|G\| = 2^8.3^8.5^2.41$			
$G = \langle a, b : a^2 = b^4 = (ab)^{41} = (ababbb)^5 = 1 \rangle$			
$S_4(9)$ = <a = a, b = b>			
a can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/clas/S49/gap/S49G1-p820aB0.g1 .			
and b can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/clas/S49/gap/S49G1-p820aB0.g2 .			
G = The Symplectic group $S_6(2)$	**$S_6(2)$ has no Hall π-subgroups**		
$\|G\| = 2^9.3^4.5.7$			
$G = \langle a, b : a^2 = b^7 = (ab)^9 = 1 \rangle$			
$S_6(2)$ = <a = bin1, b = bin1>			
bin1 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/clas/S62/gap/S62G1-p28B0.g1 .			
and bin1 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/clas/S62/gap/S62G1-p28B0.g2 .			

Appendix 4 (Cont.)

G = The Unitary group $U_3(3)$	**$U_3(3)$ has no Hall π-subgroups**		
$\|G\| = 2^5.3^3.7$			
$G = \langle a, b : a^2 = b^6 = (ab)^7 = 1$			
$U_3(3)$ = <a = bin1, b = bin1>			
bin1 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/clas/U33/gap/U33G1-p28B0.g1.			
and bin1 can be obtained from:			
http://brauer.maths.qmul.ac.uk/Atlas/clas/U33/gap/U33G1-p28B0.g2 .			
G = The Unitary group $U_3(4)$	{2,3}	The Hall {2,3}-subgroup M of $U_3(4)$ is of order 192. M can be generated by: M = <(3,4,5)(6,10,14)(7,11,15)(8,12,16)(9,13,17)(19,21,20)(22,30,26)(23,31,27)(24,32,28)(25,33,29)(34,50,66)(35,53,68)(36,51,69)(37,52,67)(38,62,74)(39,63,75)(40,64,76)(41,65,77)(42,54,78)(43,55,79)(44,56,80)(45,57,81)(46,58,70)(47,59,71)(48,60,72)(49,61,73)(82,146,210)(83,149,212)(84,147,213)(85,148,211)(86,158,218)(87,159,219)(88,160,220)(89,161,221)(90,150,222)(91,151,223)(92,152,224)(93,153,225)(94,154,214)(95,155,215)(96,156,216)(97,157,217)(98,162,226)(99,165,228)(100,163,229)(101,164,227)(102,174,234)(103,175,235)(104,176,236)(105,177,237)(106,166,238)(107,167,239)(108,168,240)(109,169,241)(110,170,230)(111,171,231)(112,172,232)(113,173,233)(114,178,242)(115,181,244)(116,179,245)(117,180,243)(118,190,250)(119,191,251)(120,192,252)(121,193,253)(122,182,254)(123,183,255)(124,184,256)(125,185,257)(126,186,246)(127,187,247)(128,188,248)(129,189,249)(130,194,258)(131,197,260)(132,195,261)(133,196,259)(134,206,266)(135,207,267)(136,208,268)(137,209,269)(138,198,270)(139,199,271)(140,200,272)(141,201,273)(142,202,262)(143,203,263)(144,204,264)(145,205,265),(2,8)(3,17)(4,14)(5,15)(6,13)(7,10)(9,11)(12,16)(18,230,19,233)(20,231,21,236)(22,226,25,227)(23,228,28,229)(24,237,33,235)(26,238,32,239)(27,241,29,232)(30,234,31,240)(34,221,35,219)(36,218,37,224)(3,216,41,225)(39,222,44,223)(40,214,49,217)(42,212,48,213)(43,211,45,210)(46,215,47,220)(50,261,51,260)(52,259,53,258)(54,268,57,263)(55,265,60,262)(56,271,65,270)(58,267,64,269)(59,266,61,272)(62,273,63,264)(66,127,67,126)(68,129,69,120)(70,128,73,122)(7,123,76,125)(72,117,81,116)(74,121,80,118)(75,119,77,124)(78,115,79,114)(82,243,83,242)(84,245,85,244)(86,249,89,246)(87,252,92,247)(88,257,97,248)(90,256,96,250)(91,253,93,251)(94,255,95,254)(9,167,99,172)(100,166,101,169)(102,164,105,165)(103,162,108,163)(104,170,113,176)(10,168,112,177)(107,175,109,174)(110,173,111,171)(130,161,131,152)(132,159,133,158)(134,155,137,157)(135,160,140,154)(136,147,145,146)(138,156,144,151)(139,150,141,153)(142,149,143,148)(178,202,179,208)(180,205,181,203)(182,206,185,207)(183,200,188,209)(184,199,193,204)(186,194,192,195)(187,197,189,196)(190,198,191,201),(2,17)(3,8)(4,15)(5,14)(6,11)(7,16)(9,13)(10,12)(18,223,21,216)(19,222,20,225)(22,224,28,221)(23,219,25,218)(24,213,31,210)(26,217,27,215)(29,220,32,214)(30,211,33,212)(34,228,37,227)(35,229,36,226)(38,231,44,233)(39,230,41,236)(40,238,47,241)(42,237,43,240)(45,234,48,235)(46,232,49,239)(50,122,53,123)(51,128,52,125)(54,126,60,129)(55,120,57,127)(56,119,63,121)(58,114,59,117)(61,116,64,115)(62,118,65,124)(66,268,69,262)(67,263,68,265)(7,261,76,258)(71,259,73,260)(72,272,79,269)(74,273,75,270)(77,271,80,264)(78,267,81,266)(82,158,85,161)(83,159,84,152)(86,154,92,155)(87,157,89,160)(88,148,95,147)(90,150,91,156)(93,151,96,153)(94,146,97,149)(98,208,101,205)(99,20,100,203)(102,207,108,200)(103,209,105,206)(104,204,111,198)(106,195,107,196)(109,197,112,194)(110,201,113,199)(130,245,133,242)(131,244,132,243)(134,252,140,246)(135,249,137,247)(136,255,143,248)(138,251,139,250)(141,256,144,253)(142,257,145,254)(162,185,165,183)(163,182,164,188)(166,179,172,180)(167,181,169,178)(168,187,175,186)(170,191,171,184)(173,193,176,190)(174,192,177,189) >,	The Fusion Maps of the conjugacy classes of M into G are: 1a, 2a, 3a, 3a, 4a, 4a, 4a, 4a, 4a.
$\|G\| = 2^6.3.5^2.13$		and it is Isomorphic to the group $((C_2 \times C_2) . (C_2 \times C_2 \times C_2)):C_3$	The Induced Character of $1_M \uparrow^G$ is = 1a + 13abcd + 52abcd + 64a.
$G = \langle a, b : a^2 = b^3 = (ab)^{13} = 1$		Some properties of M:	
$U_3(4)$ = <a = bin1, b = bin1>		M is SIMPLE : false	
bin1 can be obtained from:		M is ABELIAN : false	
http://brauer.maths.qmul.ac.uk/Atlas/clas/U34/gap/U34G1-p65B0.g1 .		M is Cyclic : false	
and bin1 can be obtained from:		M is NILPOTENT : false	
http://brauer.maths.qmul.ac.uk/Atlas/clas/U34/gap/U34G1-p65B0.g2 .		M is Solvable : true	

Appendix 4 (Cont.)

	{3,13}	The Hall {3,13}-subgroup M of $U_3(4)$ is of order 39. M can be generated by: M = < (1,77,111,204,190,138,230,44,250,271,118,62,173)(2,105.238,262,226,129,261,179,200,36,188,47,55)(3,260,24,155,33,178,8,257,60,61,109,102,136)(4,26,12,210,48,40,127,175,78,206,241,240,212)(5,86,177,68,267,52,221,98,187,162,72,92,16)(6,43,133,95,154,94,243,235,11,115,247,137,197)(7,229,202,23,169,71,25,122,35,14,207,219,126)(9,168,66,24,215,249,161,152,135,272,131,164,81)(10,140,147,192,54,29,108,272,149,246,15,183,265)(13,158,116,234,248,208,82,84,128,145,42,194,159)(17,64,189,99,83,220,167,73,214,8,53,269,112)(18,231,153,171,80,176,75,151,41,20,63,93,110)(19,273,251,90,139,184,21,144,124,216,225,198,256)(22,196,134,263,89,258,218,180,87,185,252,114,28)(27,211,58,125,100,148,195,117,146,50,205,106,213)(30,88,232,51,157,101,49,97,31,37,174,79,227)(32,254,181,228,165,76,132,242,166,67,34,259,143)(38,150,223,264,270,39,233,191,193,222,156,236,91)(45,107,255,244,46,224,239,130,142,59,237,103,57)(56,253,104,170,74,65,141,113,119,121,96,199,201)(69,120,182,123,203,268,209,163,70,186,160,266,172),(1,96,91)(2,27,32)(3,40,46)(4,255,257)(5,196,194)(6,101,99)(7,183,182)(8,241,239)(10,7,71)(11,51,53)(12,142,136)(13,221,218)(14,265,268)(15,172,169)(16,114,116)(17,154,157)(18,80,171)(19,144,251)(20,176,75)(21,256,139)(22,128,267)(23,272,123)(24,240,59)(25,192,203)(26,224,155)(28,208,187)(29,160,219)(30,64,235)(31,112,43)(33,48,107)(34,226,213)(35,246,186)(36,211,228)(37,189,247)(38,204,141)(39,138,201)(41,153,63)(42,7,258)(44,65,156)(45,260,78)(47,117,166)(49,167,115)(0,254,262)(52,263,248)(54,163,202)(55,205,165)(56,193,77)(57,102,212)(58,132,129)(60,210,103)(61,12,130)(62,74,191)(66,161,249)(67,105,125)(68,252,159)(69,122,108)(73,197,232)(76,238,195)(79,214,133)(81,131,215)(82,92,87,87)(83,95,88)(84,86,89)(85,94,97)(98,185,145)(100,143,188)(104,150,230)(106,242,200)(109,206,244)(110,231,151)(111,170,270)(113,264,173)(118,253,236)(119,233,250)(120,207,147)(121,222,190)(124,184,216)(126,149,209)(134,158,162)(13,164,152)(137,227,269)(140,266,229)(146,259,179)(148,181,261)(16,245,217)(174,220,243)(175,237,178)(177,180,234)(198,273,225)(199,223,271)>,	The Fusion Maps of the conjugacy classes of M into G are: 1a, 3a, 13a, 3a, 13d, 13c, 13b.
		Some properties of M:	The Induced Character of $1_M \uparrow^G$ is = 1a + 13abcd + 39ab + 52aabbccdd + 64aa + 65abcde + 75aabbccdd.
		M is SIMPLE : false	
		M is ABELIAN : false	
		C_{13} M is Cyclic : false	
		C_3 M is NILPOTENT : false	
		M is Solvable : true	

Appendix 4 (Cont.)

	{3,5}	The Hall {3,5}-subgroup M of $U_3(4)$ is of order 75 . M can be generated by: M = <(1,273,270)(2,30,33)(3,105,103)(4,83,84)(5,70,76)(6,200,207)(7,164,163)(8,235,234)(9,58,59)(10,143,136)(11,247,249)(12,34,37)(13,157,160)(15,220,214)(16,192,189)(17,117,114)(18,225,110)(19,193,62)(20,65,190)(21,113,222)(22,257,46)(23,81,126)(24,145,206)(25,129,78)(26,177,158)(27,161,174)(28,49,254)(29,241,94)(31,209,142)(32,97,238)(35,155,102)(36,108,154)(38,231,56)(39,144,80)(40,169,212)(41,77,141)(42,178,95)(43,88,181)(44,63,233)(45,122,195)(47,211,167)(48,196,123)(50,125,168)(51,112,137)(52,135,109)(53,175,128)(54,148,79)(55,210,203)(57,202,213)(60,72,147)(61,85,246)(64,252,82)(66,106,239)(67,194,243)(68,244,197)(69,232,107)(71,187,162)(73,165,186)(74,96,216)(75,223,93)(86,127,100)(87,172,133)(89,130,166)(90,151,199)(91,201,153)(92,99,120)(98,18,208)(101,205,182)(104,256,170)(111,171,253)(115,217,240)(116,237,215)(118,138,250)(119,150,184)(121,191,156)(124,251,139)(131,229,149)(132,146,226)(134,219,179)(140,180,218)(152,221,242)(159,245,224)(173,204,236)(176,230,198)(183,227,255)(185,248,228)(258,265,262)(259,266,269)(260,272,267)(261,268,263),(1,273,190)(2,30,33)(3,157,215)(4,117,68)(5,58,140)(6,70,127)(7,14,83)(8,220,154)(9,192,267)(10,105,248)(11,200,169)(12,164,197)(13,235,48)(15,143,54)(16,247,109)(17,34,226)(18,225,222)(19,193,270)(20,65,110)(21,113,62)(22,257,206)(23,81,238)(24,145,126)(25,129,142)(26,177,46)(27,161,94)(28,49,174)(29,241,78)(31,209,254)(32,97,158)(35,60,214)(36,79,42)(37,131,194)(38,184,264)(39,199,144)(40,221,52)(41,91,253)(43,102,101)(44,121,153)(45,160,115)(4,245,87)(50,203,88)(51,167,249)(53,57,240)(55,116,123)(56,74,151)(59,179,266)(61,242,166)(63,93,204)(64,159,66)(67,84,163)(69,246,219)(71,172,82)(72,181,268)(73,130,106)(75,201,77)(76,99,134)(80,216,256)(85,186,269)(86,89,212)(90,104,119)(92,133,232)(95,128,98)(96,230,138)(100,218,239)(103,227,217)(107,162,152)(108,208,196)(111,156,173)(112,189,259)(114,243,229)(118,176,170)(120,207,211)(122,213,205)(124,236,251)(12,182,258)(132,244,146)(135,187,260)(136,147,255)(137,165,224)(139,223,171)(148,185,262)(150,198,231)(155,234,195)(168,228,237)(175,183,261)(178,265,202)(180,272,252)(188,263,210)(191,271,233)>,	The Fusion Maps of the conjugacy classes of M into G are: 1a, 5c, 5b, 5d, 5a, 5e, 5f, 5f, 5e, 3a, 3a.
		and it is Isomorphic to the group $(C_5 \times C_5) : C_3$	The Induced Character of $1_M \uparrow^G$ is = 1a + 13abcd + 39ab + 52abcd + 64aa + 65a + 75abcd.
		Some properties of M:	
		M is SIMPLE : false	
		M is ABELIAN : false	
		M is Cyclic : false	
		M is NILPOTENT : false	
		M is Solvable : true	

Appendix 4 (Cont.)

G = The Unitary Group $U_3(5)$	
$\lvert G \rvert = 2^4.3^2.5^3.7$	
$G = \langle a, b : a^3 = b^5 = (ab)^7 = 1$	
$U_3(5)$ = <a = MeatAxe.Perms, b = MeatAxe.Perms>	**$U_3(5)$ has no Hall π-subgroups**
MeatAxe.Perms can be obtained from:	
http://brauer.maths.qmul.ac.uk/Atlas/clas/U35/gap/U35G1-p50B0.g1 .	
and MeatAxe.Perms can be obtained from:	
http://brauer.maths.qmul.ac.uk/Atlas/clas/U35/gap/U35G1-p50B0.g2 .	

Cite this article as: Sarah Mohammed Abdullah Alhwaimel (2022). The Hall π-Subgroups of Some of the Classical Simple Groups. *International Journal of Pure and Applied Mathematics Research*, 2(2), 63-74. doi: 10.51483/IJPAMR.2.2.2022.63-74.